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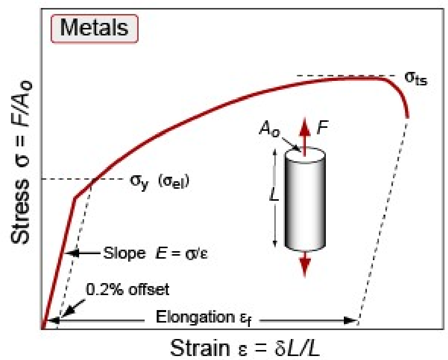
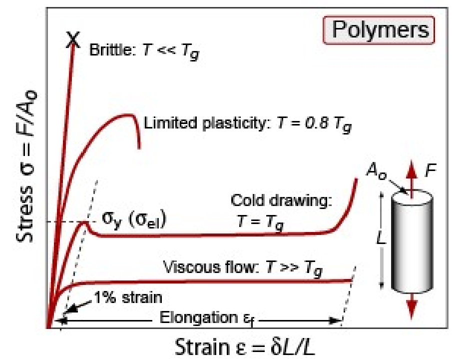
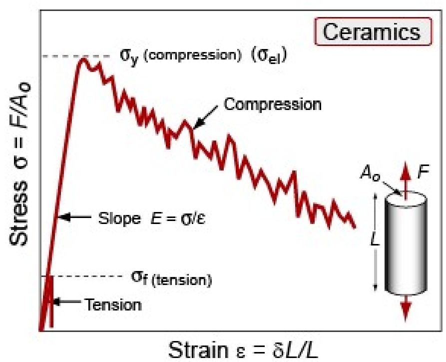
Material properties: static strength

Yield strength, elastic limit and ultimate strength







Definition of yield strength:

metals: 0.2% offset yield strength
polymers: 1% offset
polymer composites: 0.5% offset
ceramics and glasses: compressive strength

Note the difference in the yielding of metals, polymers and ceramics.

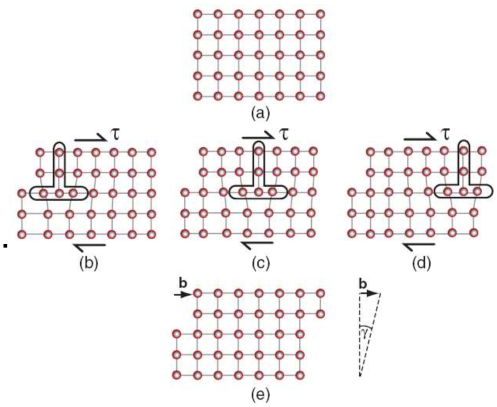
Metals work-harden, polymers are brittle at RT. Ceramics fracture before yielding.

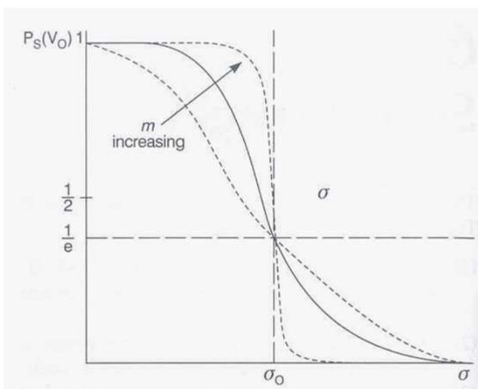
Origin of strength - yield strength limits

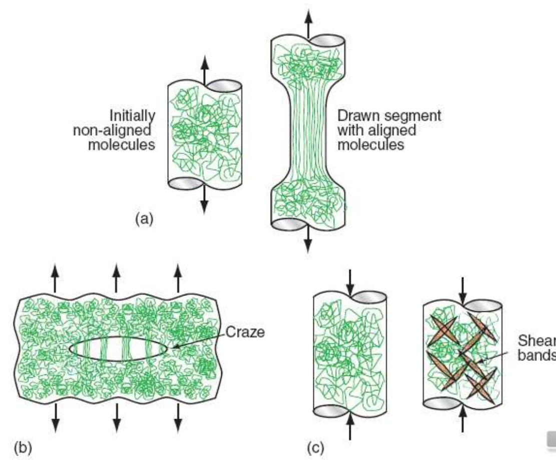
Range:
Theoretical strength: E/10
In applications: 0.01 MPa (foams) to 10GPa (diamond)

Mechanisms:
glide of dislocations in metals, slip of polymer chains, …
depends on temperature

Variability in strength:
Weibull statistics $P(V)=\exp(-\sigma/\sigma_0)^m$)



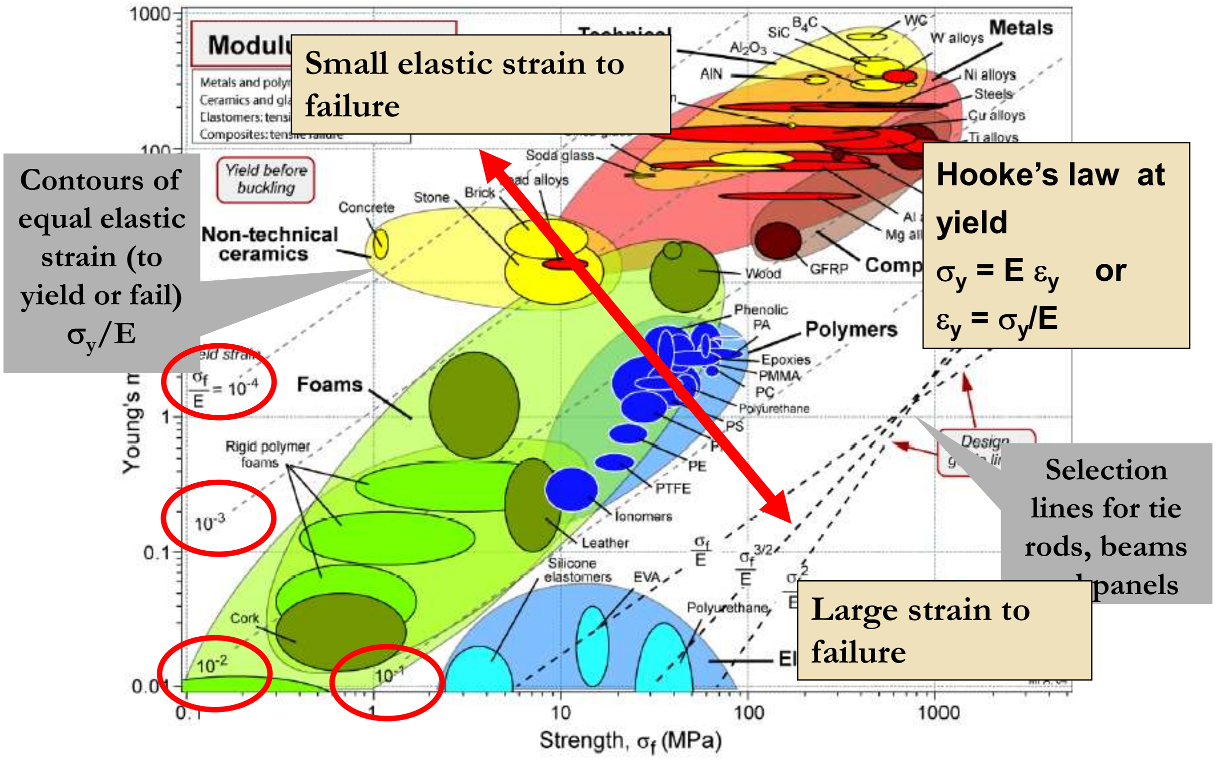




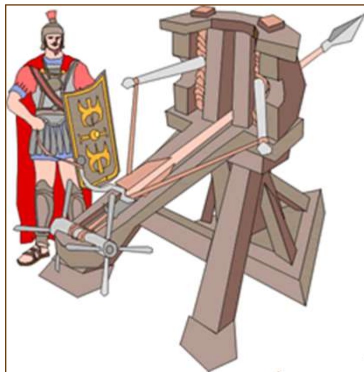
Taylor and Orowan realized that dislocations enable deformation and cause yielding in materials much below their theoretical strenghts.

The statistics of strength are reflected in the Weibull modulus m, which indicated the proportion of similar samples of the same volume V_0 which will survive loading to a given level of stress σ. m the Weibull modulus and s0 are constants. When $\sigma=\sigma_0$ then a fraction 1/e, e.g. 37% will survive the loading.

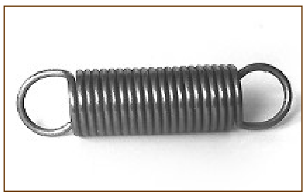
Young's modulus vs Strength

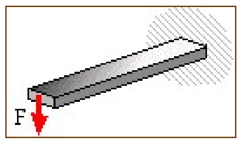


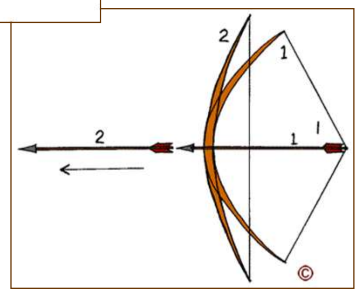
Materials for springs

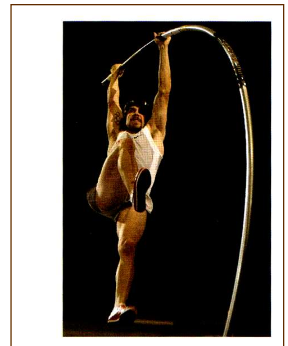


Images from: http://www.mech.uwa.edu.au/DANotes/springs/intro/intro.html
http://www.ftexploring.com/lifetech/flsbws2.html









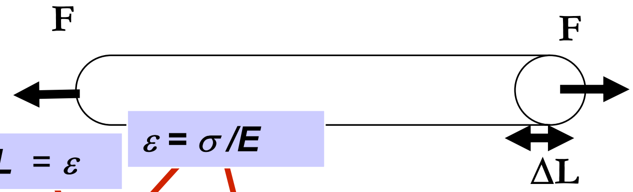
A pole vaulter – the pole stores elastic energy.

Small springs: minimum volume
Light springs: minimum mass

Material for a spring of minimum volume
Cross section of given shape

Goal: minimise V for given amount of elastic energy stored, W

Volume $V = LA \qquad A = V/L$



$F = \sigma A$ $A = V/L$ $\Delta L/L = \varepsilon$ $\varepsilon = \sigma / E$

Elastic energy *W*

$$W = \tfrac{1}{2} F \Delta L = \tfrac{1}{2}\sigma A\, \Delta L = \tfrac{1}{2}\sigma V\, \Delta L/L = \tfrac{1}{2}\sigma\varepsilon V = \tfrac{1}{2} V \sigma^2/E$$

Solving for V:

$$V = 2W \frac{E}{\sigma^2}$$

minimise V for given W….?

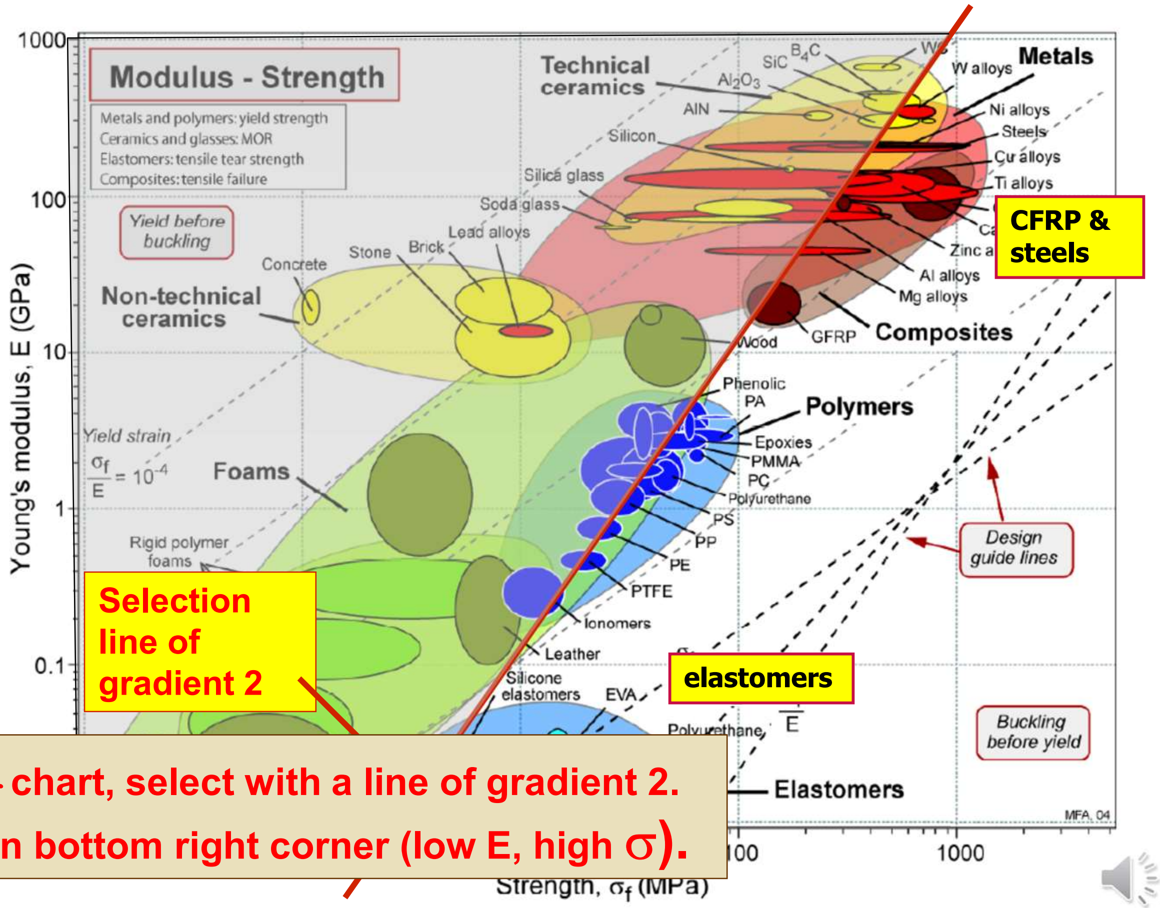
maximise

$$M_{small} = \frac{\sigma^2}{E}$$

On E - σ_y or E - σ_f chart, select with a line of gradient 2. Search on bottom right corner.

spring of minimum volume

$M = \frac{\sigma^2}{E}$



On E - σ_f chart, select with a line of gradient 2.

Search on bottom right corner (low E, high σ).

Material for a spring of minimum mass

Cross section of given shape

For minimum Volume V

$$M_{small} = \frac{\sigma^2}{E}$$

For minimum mass m = ρV

$$V = 2W\frac{E}{\sigma^2}$$

Three materials properties in a single index: separate ?

$$M_{light} = \frac{\sigma^2}{E\rho}$$

$$m = \rho V = 2W\frac{\rho E}{\sigma^2}$$

Solving M_{light} for E:

$$E = \frac{\sigma^2}{M\rho} = \frac{\sigma^2}{M\rho}\frac{\rho}{\rho} = \frac{\rho}{M}\frac{\sigma^2}{\rho^2}$$

$$\frac{E}{\rho} = \frac{1}{M}\left(\frac{\sigma}{\rho}\right)^2$$

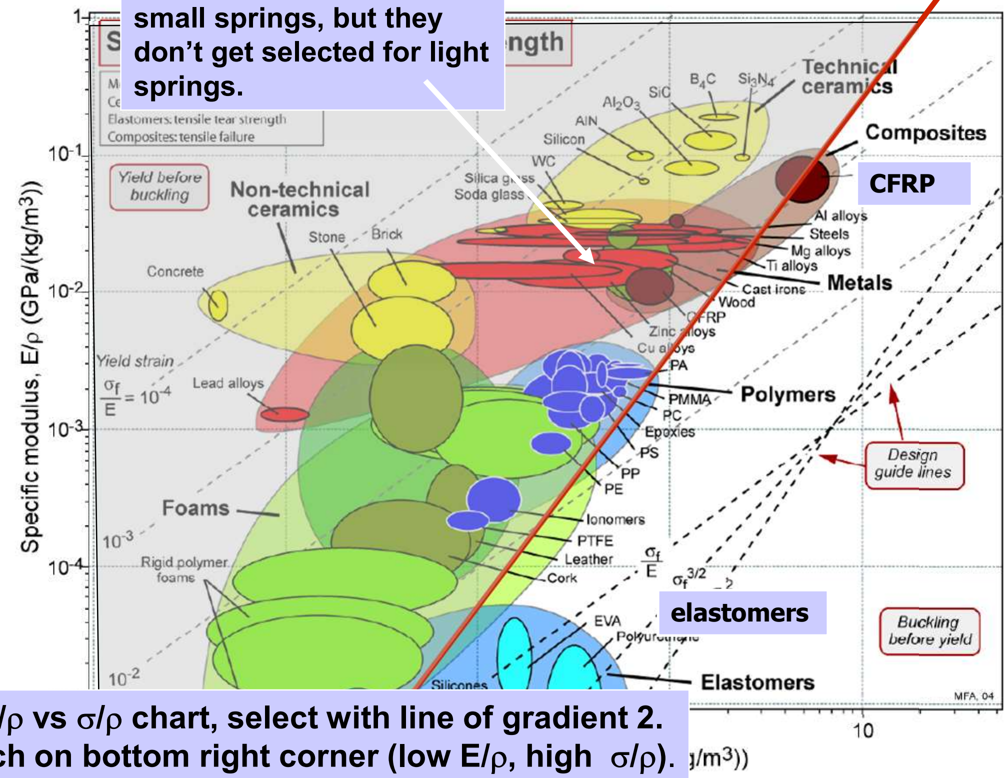
On E/ρ vs σ/ρ chart, select with line of gradient 2.

Search on bottom right corner (low E/ρ, high σ/ρ).

...imum mass

$M = \frac{(\sigma / \rho)^2}{E / \rho}$

Metals moved back and down. They are good for small springs, but they don't get selected for light springs.



CFRP

elastomers

On E/ρ vs σ/ρ chart, select with line of gradient 2.
Search on bottom right corner (low E/ρ, high σ/ρ).

Table B.3 Strength-limited design: springs, hinges, etc. for maximum performance

Function and constraints	Maximize
Springs	
Maximum stored elastic energy per unit volume; no failure	σ_f^2/E
Maximum stored elastic energy per unit mass; no failure	$\sigma_f^2/E\rho$
Elastic hinges	
Radius of bend to be minimized (max flexibility without failure)	σ_f/E
Knife edges, pivots	
Minimum contact area, maximum bearing load	σ_f^3/E^2 and H
Compression seals and gaskets	
Maximum conformability; limit on contact pressure	$\sigma_f^{3/2}/E$ and $1/E$
Diaphragms	
Maximum deflection under specified pressure or force	$\sigma_f^{3/2}/E$
Rotating drums and centrifuges	
Maximum angular velocity; radius fixed; wall thickness free	σ_f/ρ

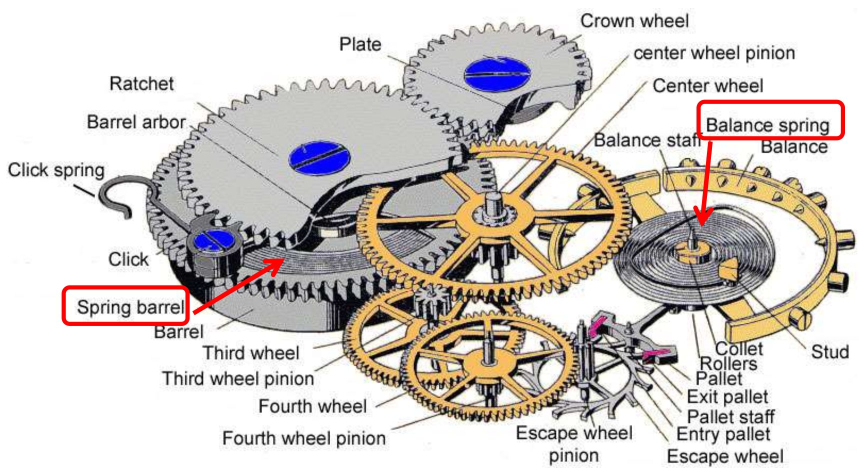
Case study
Springs for mechanical watches

www.piaget.fr

How does a mechanical watch work? link

There are two major springs:

- *Mainspring (or spring barrel)*
- *Hairspring (or balance / spiral spring)*



A hairspring (or balance spring) is a tightly spiraled spring attached to the balance wheel. Together with the balance wheel, a harmonic oscillator is formed – put simply, a system which experiences a restorative force equal to that of the displacement force. In a vacuum, a harmonic oscillator is perfect and could continue to oscillate forever, but, this is the real world, and things like friction get in the way. With the help of regulation and a supply of power from the mainspring, the hairspring and balance wheel can overcome external forces and run at a precise resonant frequency.

Current fabrication process of hairspring: (1) Thin rolled metal sheets cut into ribbons. (2) Metal ribbons are coiled to form a spiral. (3) Production requires know-how and manual adjustements.

Case study
Springs for mechanical watches

What is a mainspring?

- *provides energy to the mechanism by storing and releasing energy upon winding and unwinding of the spring.*
- *One component of the oscillator with the balance wheel.*
- *Manufacturing process video: link*

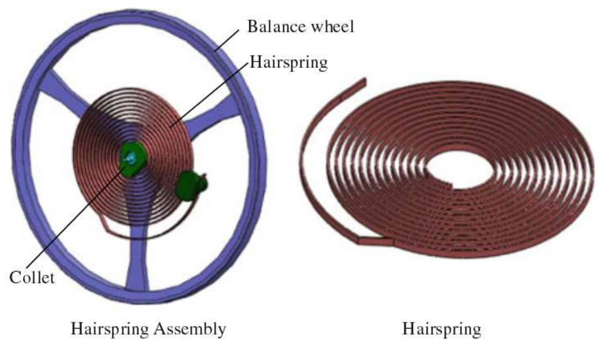
www.mauricelacroix.com

Mechanical pocket watches and wristwatches have traditionally been driven by a leaf spring, which is the barrel spring or main spring, wound inside a barrel drum. The external part of the spring presses against the inside wall of the drum and one end is fastened to the drum. The inner end of the spring is fastened to a barrel arbor. By keeping the drum fixed and rotating the barrel arbor, the spring is wound around the arbor and potential (strain) energy is accumulated in the spring.

Case study
Springs for mechanical watches

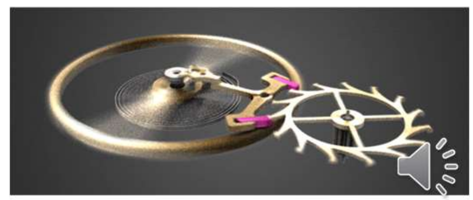
What is a spiral hairspring?

- *One component of the oscillator with the balance wheel.*
- *provides time regulation through periodic oscillation.*
- *Manufacturing process video: link*



DOI 10.1007/978-3-642-29308-5

www.mauricelacroix.com



A hairspring (or balance spring) is a tightly spiraled spring attached to the balance wheel. Together with the balance wheel, a harmonic oscillator is formed – put simply, a system which experiences a restorative force equal to that of the displacement force. In a vacuum, a harmonic oscillator is perfect and could continue to oscillate forever, but, this is the real world, and things like friction get in the way. With the help of regulation and a supply of power from the mainspring, the hairspring and balance wheel can overcome external forces and run at a precise resonant frequency.

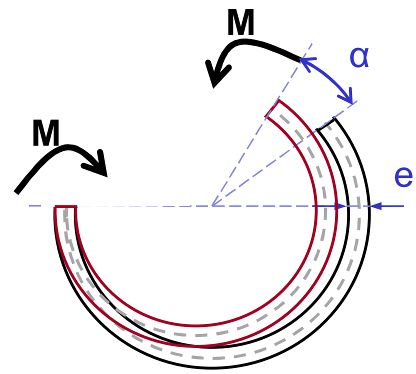
Current fabrication process of hairspring: (1) Thin rolled metal sheets cut into ribbons. (2) Metal ribbons are coiled to form a spiral. (3) Production requires know-how and manual adjustements.

Materials selection for spiral hairspring

Objective *Maximize the stored elastic energy, w*

Energy stored in torsion spring:

$$W = \frac{2IL}{e^2}\frac{{\sigma_y}^2}{E} = \frac{heL}{6}\frac{{\sigma_y}^2}{E}$$



Variables

- *Choice of material*

(Dimensions are fixed)

Performance index

$$W = \frac{heL}{6}\left(\frac{{\sigma_y}^2}{E}\right)$$

Choose materials with largest: $\frac{{\sigma_y}^2}{E}$

The energy stored in the spring will be maximum when the deformation angle is maximum.

In that case, we can calculate the maximum bending angle as a function of the spring length L, thickness e and the mechanical properties of the spring (i.e. E and sigma_y).

Modulus-Strength Chart

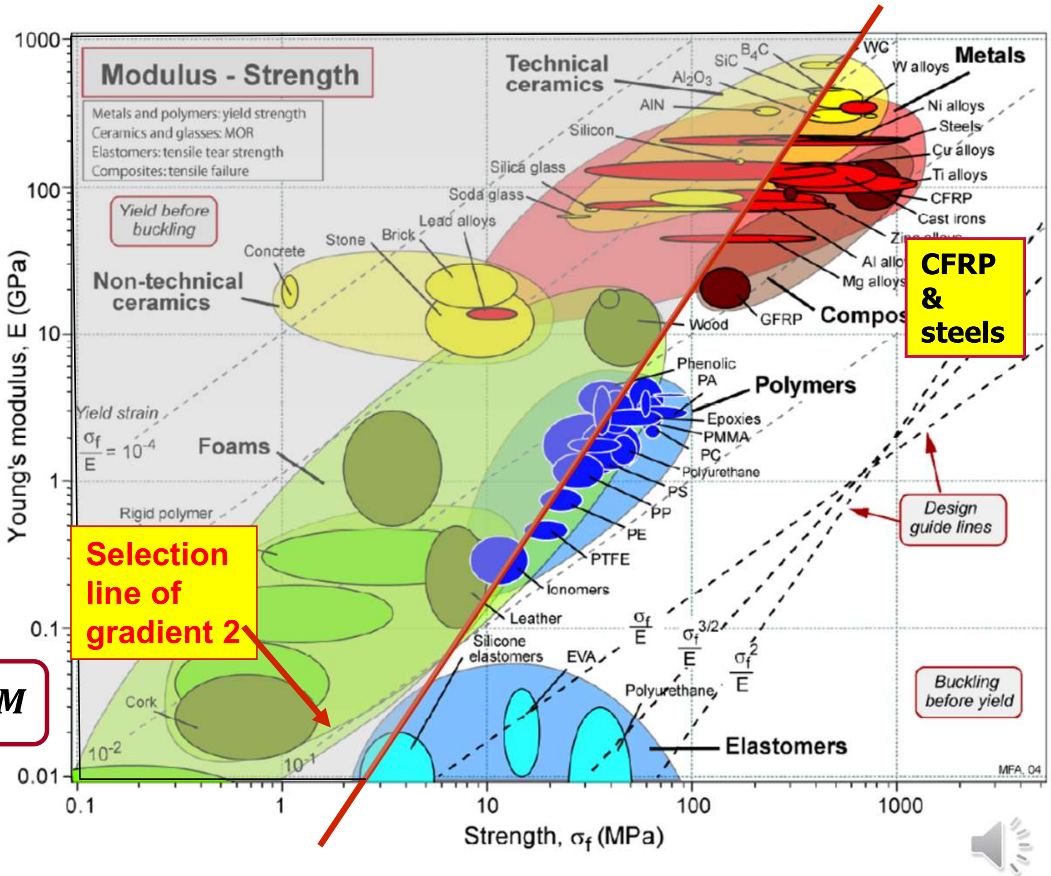
Index $M = \frac{\sigma_y^{\,2}}{E}$

Rearrange:

$E = \sigma_y^{\,2}/M$

Take logs:

$\log E = 2 \log \sigma_y - \log M$



Performance index for several materials

$$M = \frac{\sigma_y{}^2}{E}$$

Materials for Efficient Small Springs

Material	$M_1 = \frac{\sigma_f^2}{E}$ (MJ/m^3)	Comment
Ceramics	(10 – 100)	Brittle in tension; good only in compression.
Spring steel	10	The traditional choice; easily formed and heat treated.
Ti alloys	10	Expensive, corrosion-resistant.
CFRP	8	Comparable in performance with steel; expensive.
GFRP	5	Almost as good as CFRP and much cheaper.
Glass	10	Brittle in torsion, but excellent if protected against damage; very low loss factor.
Nylon	3	The least good; but cheap and easily shaped, but high loss factor.
Rubber	20	Better than spring steel; but high loss factor.

Materials Selection in Mechanical Design, Ashby

For timekeeping device, accuracy is key. That is the reason why materials with loss factor as small as possible are preferred. For springs, we usually use low-carbon steels or alloys based on Nickel, Cobalt or Titanium (e.g. Elgiloy, a Cr-Co-Ni-Fe-Mo-based alloy, or Nivarox, a Fe-Ni-Cr-Ti-Al-alloy)

Metals used for hairspring – Nivarox & Silicon

What is actually used in the watch industry?

NIVAROX:

As a trade name, Nivarox is a German acronym for "Nicht variabel oxydfest" or "Non-Variable Non-Oxidizing" (E.). The Nivarox alloy is a nickel iron alloy for hairsprings for balance wheels, in the same category as Elinvar, Ni-Span, Vibralloy and other similar.

The "non-variable" refers to the alloy's most notable property: that it has a low temperature coefficient of elasticity; its elasticity does not change much with temperature, There are several versions of the Nivarox alloy depending upon the intended application. A typical composition would be for the early version Nivarox-CT (by wt %) : Fe 54%, Ni 38%, Cr 8%, Ti 1%, Si 0.2%, Mn 0.8%, Be 0.9%, C < 0.1%.[3]

When used for critical watch components, the alloy reduces errors due to temperature variation. Hairsprings made of this alloy have a spring constant which does not vary with temperature, allowing the watch's balance wheel, its timekeeping element, to keep better time.

SILICON:

Silicon hairsprings are made by deep reactive ion etching (DRIE) and are coated by a thick silicon dioxide layer to compensate for thermal expansion. Complex shapes with high precision are possible such non uniform cross-section.

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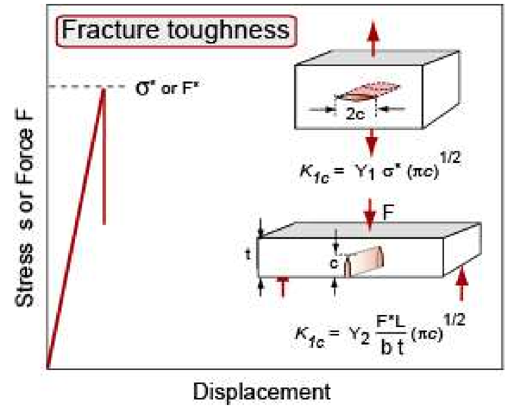
Material properties: fracture toughness

Definition:

Fracture toughness $$K_{1c} = Y_1 \sigma^* \sqrt{\pi c}$$

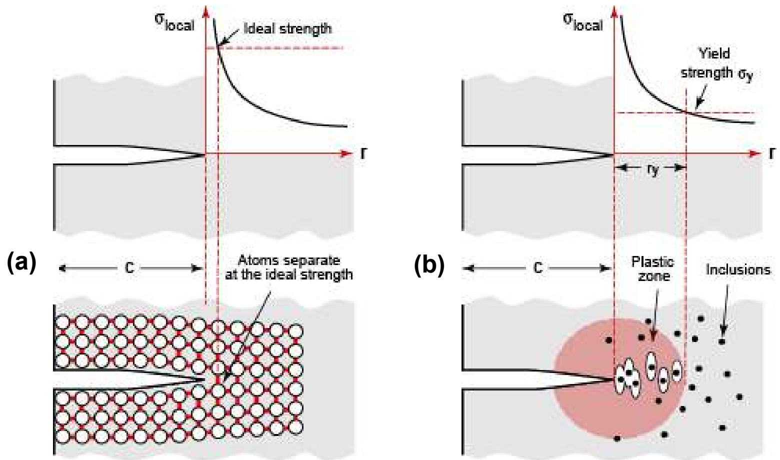
Fracture energy or toughness $$G = \frac{K_c^{\,2}}{E}$$



Range of fracture toughness:

0.3 MPa $m^{1/2}$ (glass) to 200 MPa $m^{1/2}$ GPa (steel)

Origin:



$$r_y = \frac{K_C^{\,2}}{2\pi\sigma_f^{\,2}}$$

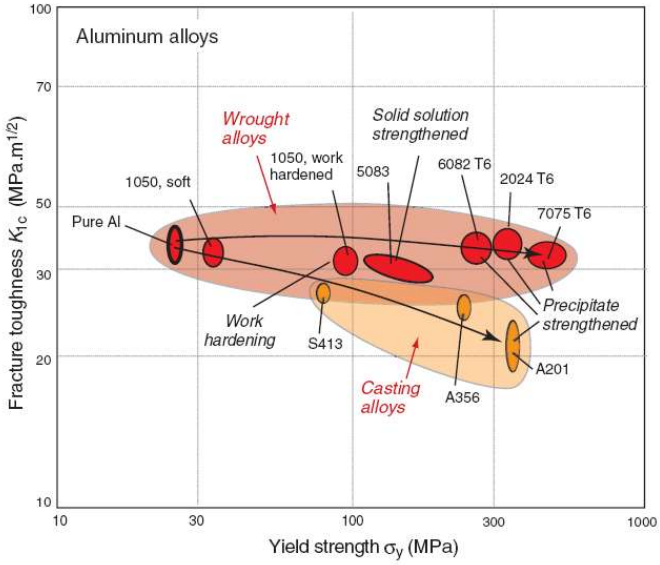
Fracture toughness: material property; independent of test geometry and usable in design

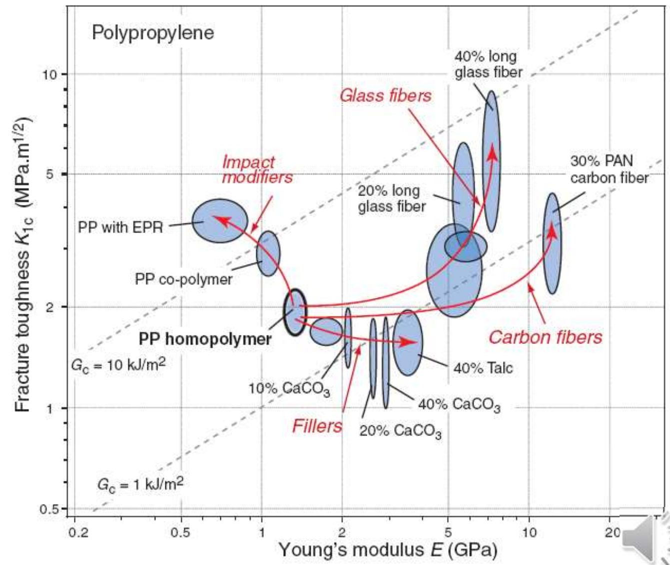
Fracture energy or Toughness: resistance to crack-propagation

(a) The local stress rises as 1 / √r towards the crack tip. If it exceeds that required to break inter-atomic bonds (the "ideal strength") they separate, giving a cleavage fracture

(b) If the material is ductile a plastic zone (r_y) forms at the crack tip. Within it voids nucleate, grow and link, advancing the crack in a ductile mode.

Manipulating strength-toughness trade-off





It is difficult to make materials both strong and tough.

Metals: Toughness is increased with no loss of strength for instance if inclusions are removed, delaying the nucleation of the voids.

Polymer composites can be toughened by reinforcement with ‘brittle’ glass or carbon fibers.

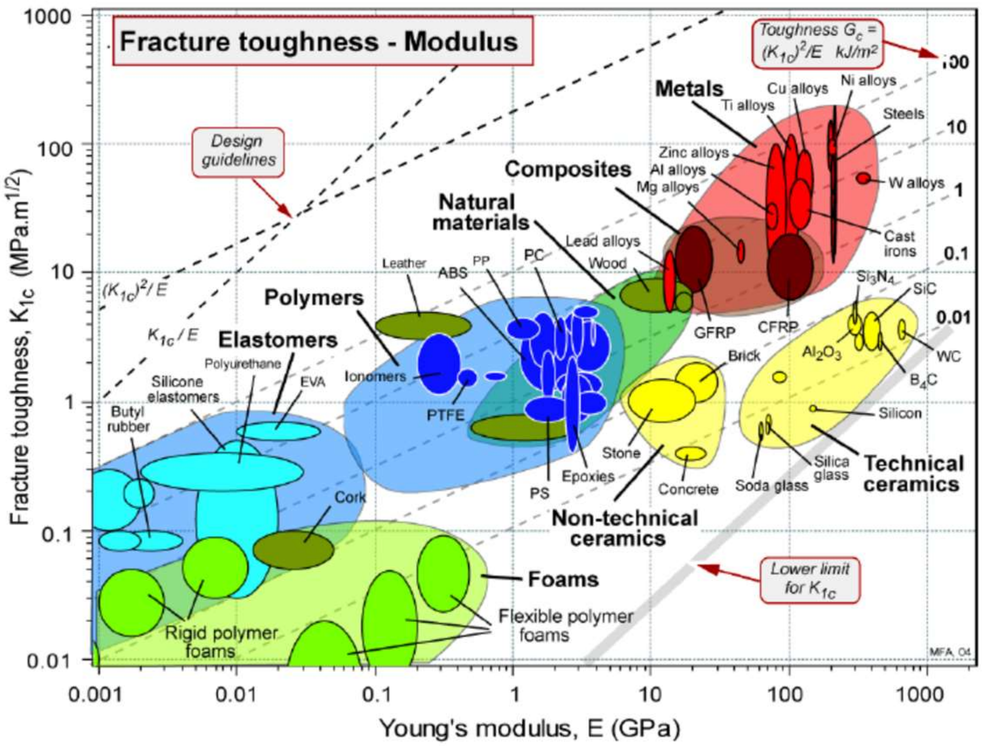
Fracture toughness vs Young's modulus

Why the differences?

- ***lower limit: surface energy***

Manipulating properties

- ***Making composites***
- ***Making foams***



The chart displays both the fracture toughness, and (as contours) the toughness. It allows criteria for stress and displacement-limited failure criteria (and E/K) to be compared. The guidelines show the loci of points for which

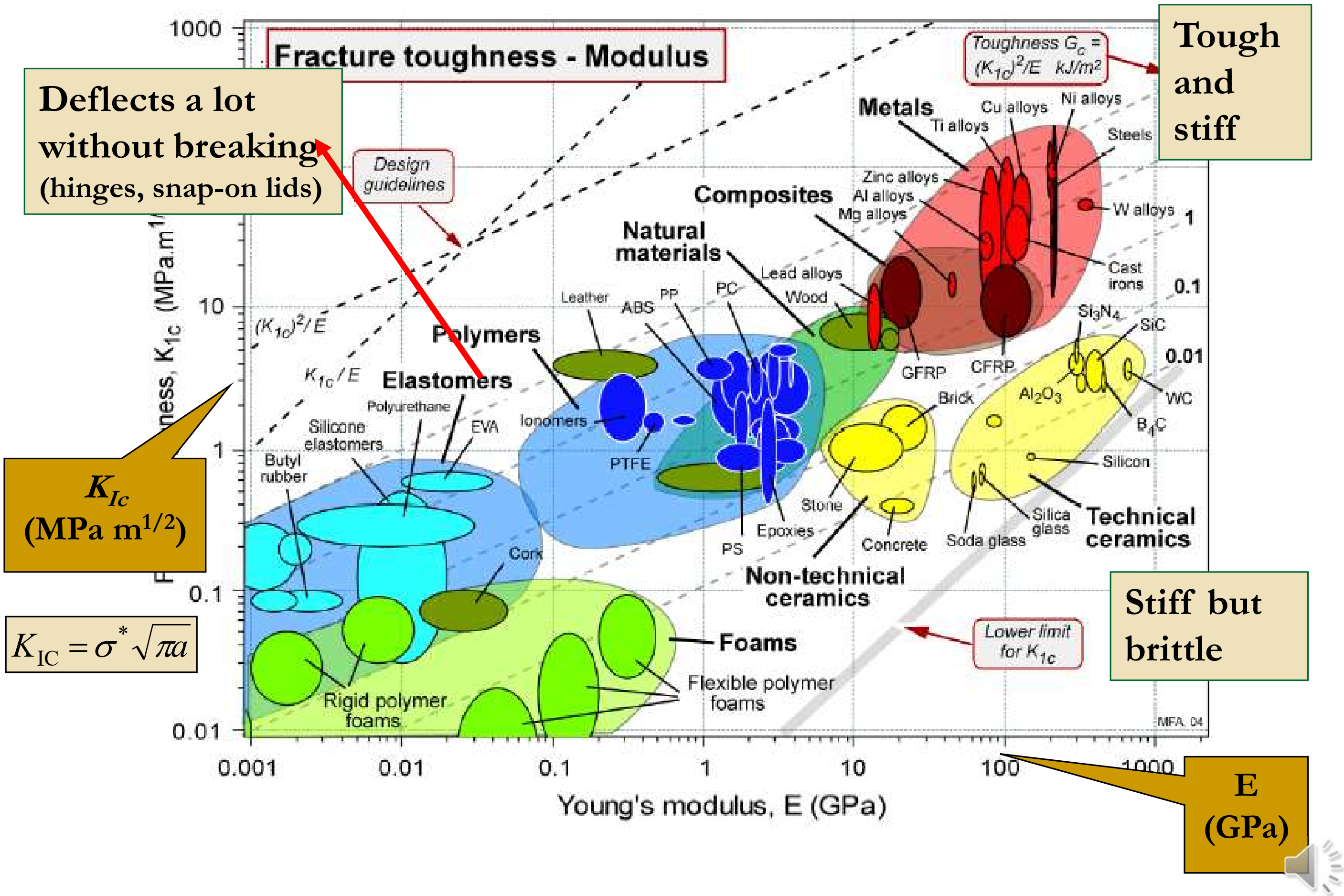
$G_{Ic} = K_{Ic}^2 / E$

(a) $K_{Ic}^2 / E = C$ (lines of constant toughness, G_c; energy-limited failure)

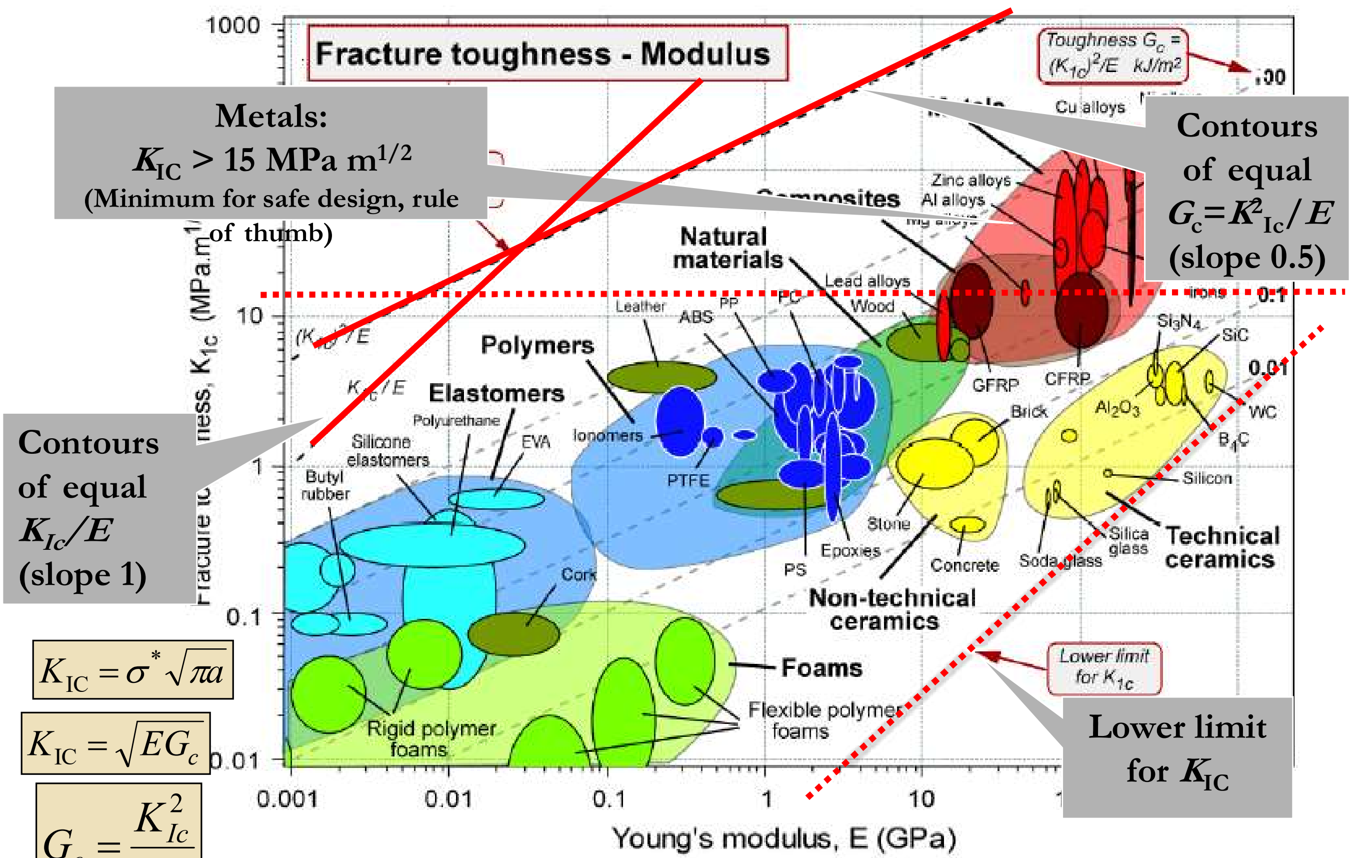
(b) $K_{Ic} / E = C$ (guideline for displacement-limited brittle failure)

The values of the constant C increases as the lines are displaced upwards and to the left. Tough materials lie towards the upper left corner, brittle materials towards the bottom right.

Fracture toughness vs Young's modulus



Fracture toughness vs Young's modulus



$$K_{IC} = \sigma^* \sqrt{\pi a}$$

$$K_{IC} = \sqrt{E G_c}$$

$$G_c = \frac{K_{Ic}^2}{E}$$

Contour lines in K_{Ic}- E chart

4 lines of interest in the K_{Ic}- E chart:

Lower limit for K_{Ic} ?

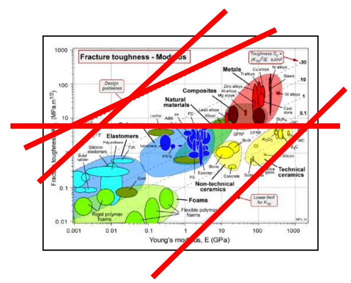
Contour lines of constant K_{Ic} ?

Contour lines at constant K_{Ic}^2/E ?

Contour lines at constant K_{Ic}/E ?

Next slide

3 Case studies



Lower limit to K_{Ic}

$$K_{IC} = \sqrt{EG_c}$$

$$\gamma = \frac{Er_o}{20}$$

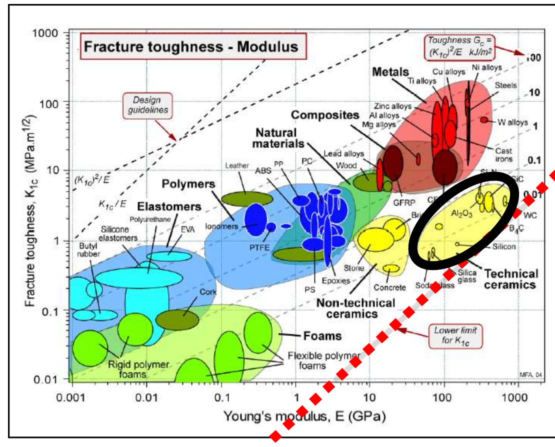
$$G_c \geq 2\gamma \Rightarrow K_{Ic} = \sqrt{2E\gamma}$$

$r_o = 2 \times 10^{-10}$ m (interatomic spacing)

$$K_{Ic} = E\left[\frac{r_o}{10}\right]^{1/2} = 3x10^{-6} m^{1/2} E$$

Lower limit for perfectly brittle materials

Ceramics & glasses nearly touch the boundary



Contour lines: Case studies in K_{Ic}-E

Three case studies:

Load limited design (component should take specified load – need to maximise the load , e.g.: tension members in cantilever bridge)

Displacement limited design (Component must deflect a given amount w/o failure – maximise deflection case, e.g.: bottle snap-on lids)

Energy absorption controlled design (component must absorb specified amount of energy prior to failure – maximise absorbed energy, e.g.: car bumper)

Contour lines: Case studies in K_{Ic}-E

Design requirements

Function	*What does the component do ?*	***A tension tie***
Constraints	*What essential conditions must it meet ?*	***A crack of a given length***
Objectives	*What measure of performance is to be maximized or minimized ?*	▪ ***maximise load*** ▪ ***maximaise displacement*** ▪ ***maximise absorbed energy***
Free variables	*Which design variables are free ?*	***Choice of material***

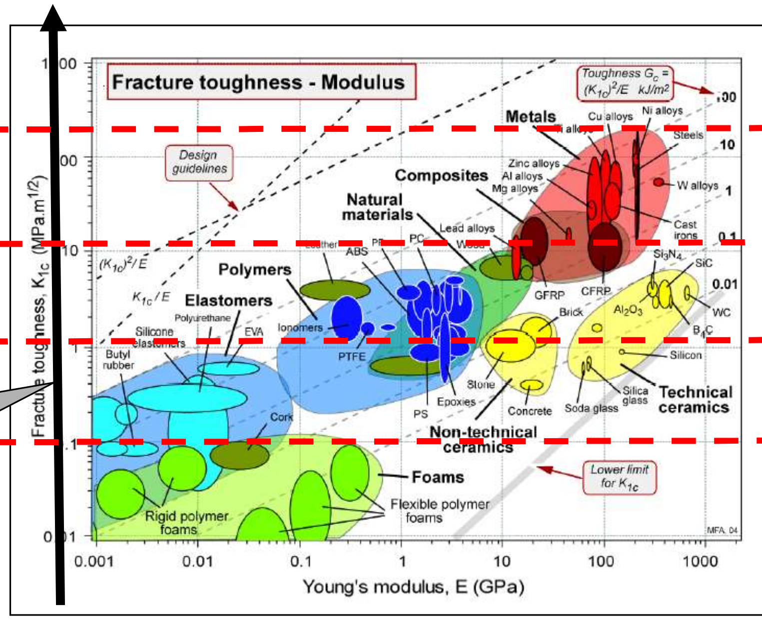
Case study 1: Load limited design (component should take specified load without failure, trivial case)

Application: tension members in a bridge

$$K_{IC} = \sigma^* \sqrt{\pi a}$$

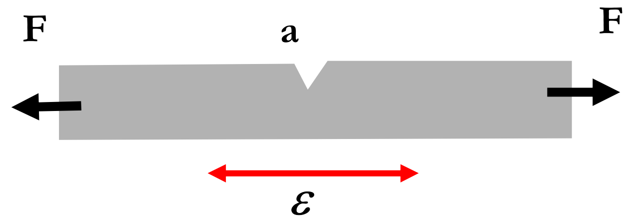
$$\sigma^* = \frac{K_{Ic}}{\sqrt{\pi a}}$$

To increase σ* for given a, increase K_{Ic}



Case study 2: Displacement limited design (Component must deflect a given amount without failure)

$$\sigma^* = \frac{K_{Ic}}{\sqrt{\pi a}}$$



Elastic strain at failure?

$\sigma^* = E\,\varepsilon^*$ **(Hooke's law)**

$$\varepsilon^* = \frac{\sigma^*}{E} = \frac{1}{E}\frac{K_{Ic}}{\sqrt{\pi a}} = const.\left[\frac{K_{Ic}}{E}\right]$$

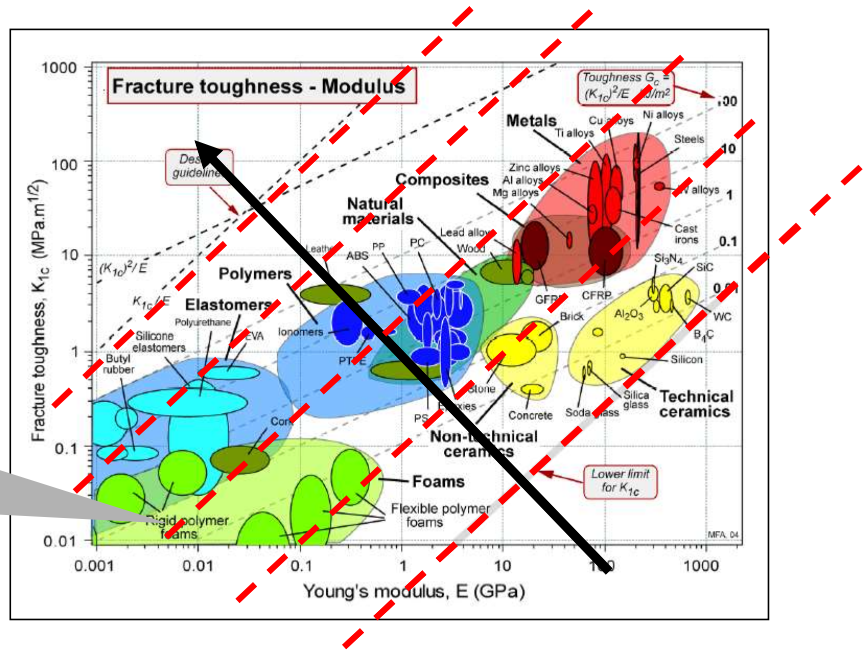
To increase ε^* for given a, increase K_{Ic}/E

$$\varepsilon^* \propto \left[\frac{K_{Ic}}{E}\right]$$

Case study 2 (cont'd.) : Displacement limited design
(Component must deflect a given amount without failure)

$$\varepsilon^* \propto \left[\frac{K_{Ic}}{E}\right]$$

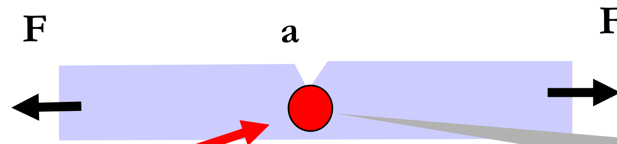
Application: plastic snap-on lids



To increase ε^* for given a, increase K_{Ic}/E

Case study 3: Energy absorption controlled design (component must absorb specified amount of energy prior to failure)

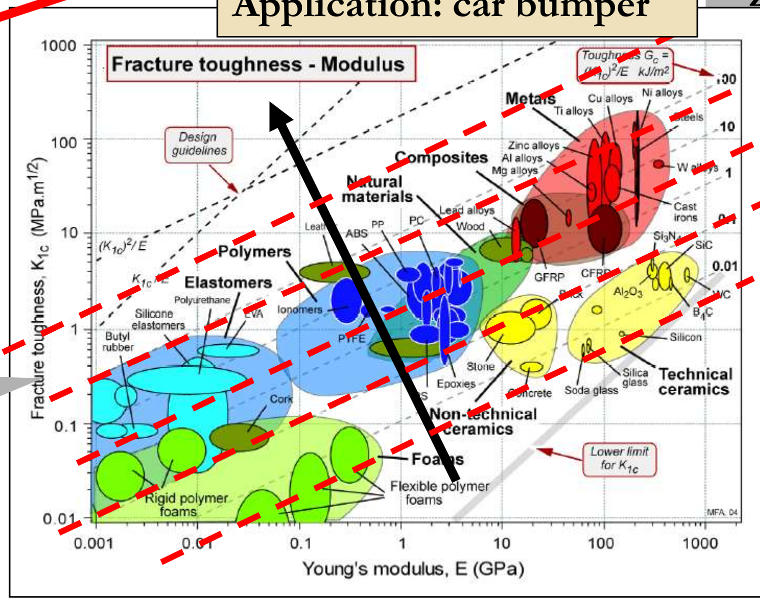
$$K_{IC} = \sqrt{EG_c}$$



Process zone

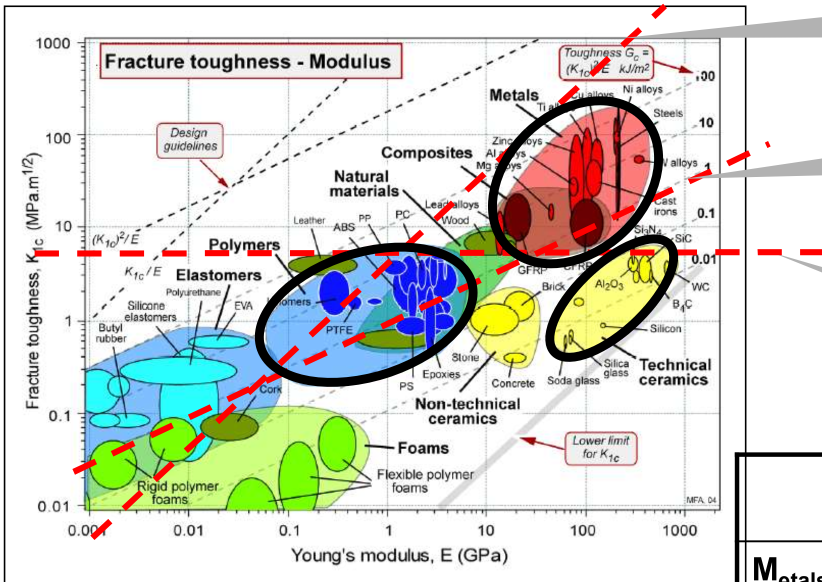
$$G_c = \frac{K_{Ic}^2}{E}$$

Application: car bumper



To increase G_c, pick materials with high $(K_{Ic})^2 / E$

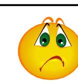
Fracture toughness vs. Young's modulus



Displacement limited design (K/E)

Energy limited design (K^2/E)

Load limited design (K)

	K	K/E	K²/E
Metals			
Polym			
Ceram			

Polymers beat ceramics despite their low K because of their very low E (K/E; K^2/E)

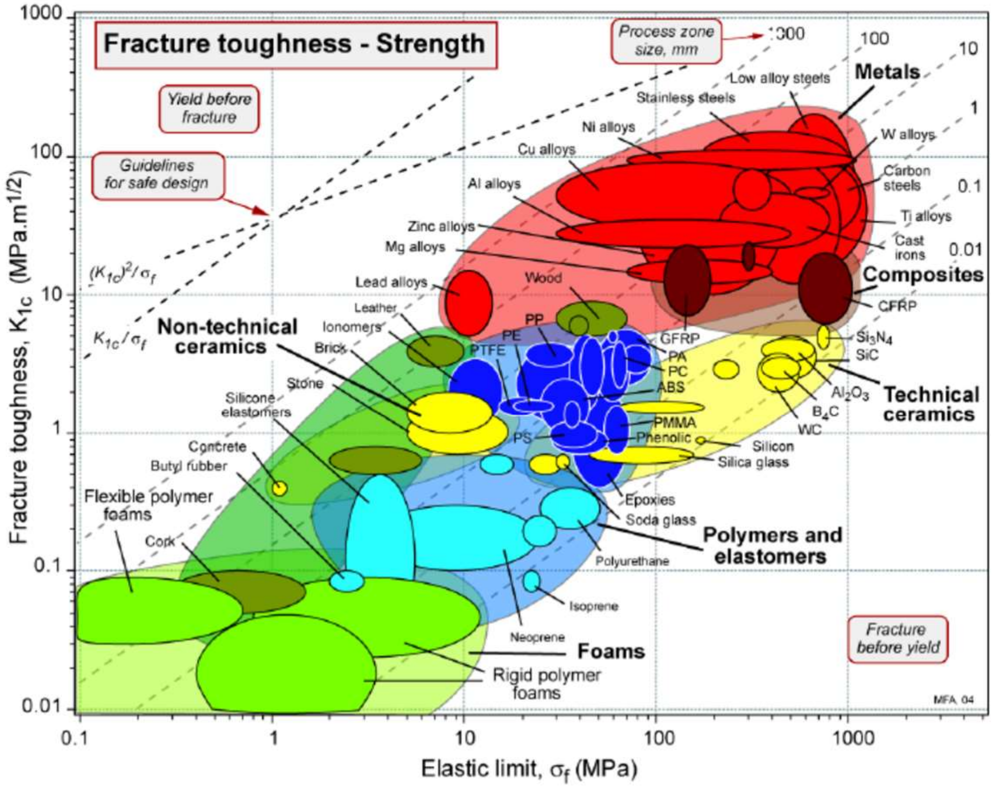
fracture toughness vs strength

Why the differences?

- ***Process zone size***

Manipulating properties

- ***Making composites***
- ***Making foams***



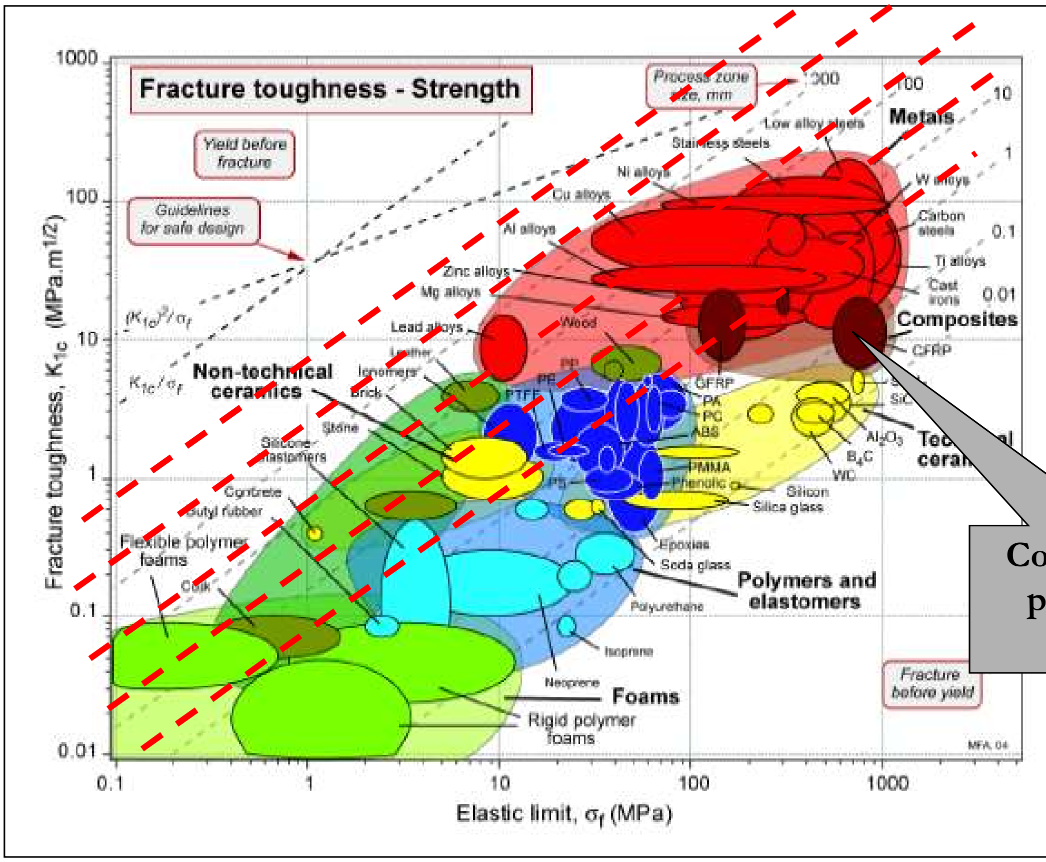
The chart for safe design against fracture. The contours show the process-zone diameter, given approximately by $K_{Ic}^2/\pi\sigma_f^2$. The qualifications on "strength" given for Charts 2 and 3 apply here also. The chart guides selection of materials to meet yield-before-break design criteria, in assessing plastic or process-zone sizes, and in designing samples for valid fracture toughness testing. The guide lines show the loci of points for which

(a) $K_{Ic}/\sigma_f = C$ (yield-before-break)

(b) $K_{Ic}^2/\sigma_f = C$ (leak-before-break)

The value of the constant C increases as the lines are displaced upward and to the left.

Fracture toughness vs. strength:
strength is important provided the material does not crack under load.



$$K_{IC} = \sigma^* \sqrt{\pi a}$$

$$a = \frac{1}{\pi}\left[\frac{K_{Ic}}{\sigma_y}\right]^2$$

Contours of equal process zone or "crack size"

Case studies in KIc- σ: Pressure vessels

Two case studies:

Yield before break, or why you can forget you coke/beer can in the freezer and nothing happens. Small vessels.

Leak before break, or why nuclear rectors don’t go bust (most of the time, anyway.) Large vessels

Case studies in KIc- σ: spherical Pressure vessels

Pressure vessels, from the simplest aerosol-can to the biggest boiler, are designed, for safety, to yield or leak before they break. The details of this design method vary. Small pressure vessels are usually designed to allow general yield at a pressure still too low to cause any crack the vessel may contain to propagate ("**yield before break**"); the distortion caused by yielding is easy to detect and the pressure can be released safely. With large pressure vessels this may not be possible. Instead, safe design is achieved by ensuring that the smallest crack that will propagate unstably has a length greater than the thickness of the vessel wall ("**leak before break**"). The leak is easily detected, and it releases pressure gradually and thus safely.

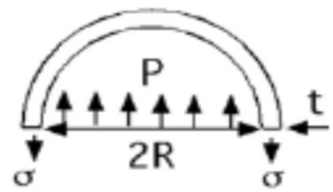
The two criteria lead to different material indices. You will be led through these in this case study.

Materials for pressure vessel

Specification

Function	Contain pressure p
Objectives	Minimum thickness Minimum weight Maximise pressure Maximise allowable crack length
Constraints	• Yield before break or • Must leak before break • Toughness adequate • Diameter $2R$ and pressure Δp specified
Free variables	• Wall thickness t • Material

Pressure vessels are pressured-limited, minimum weight, designs



We idealize the pressure vessel as a thin walled sphere

Case studies in KIc- σ: Pressure vessels

Objectives

When designing the pressure vessel the radius will be normally fixed by design (and in this case study we will consider the radius as a constraint), however there are multiple possible objectives which we can seek when we are designing pressure vessels. In this case study we will look at 4 different objectives:

Objectives:	Label:
Maximize pressure, for a given maximum crack size, before failure.	M_1
Maximize safety using leak-before-break (Objective maximise pressure under constraint leak before break)	M_2
Maximize safety using yield-before-break (Objective maximise size of safe crack under constraint ybb)	M_3
Minimize thickness	M_4
Minimize weight	M_5

ALL IN EXERCISE!

Free Variables and Constraints

These different objectives will have different free variables and constraints.

1. When designing a pressure vessel to maximize the pressure for a Maximum Flaw Size, the design will fix the radius, wall thickness and maximum flaw size, and the designer will be free to choose a material.

2&3. When maximizing safety, the radius of the pressure vessel and the pressure it will contain will be fixed, and are hence constraints, while the designer will be free to vary the wall thickness and material.

4. Minimizing wall thickness will often be considered in conjunction with maximizing safety and so will have the same constraints, and the designer will be free to choose a material.

M1

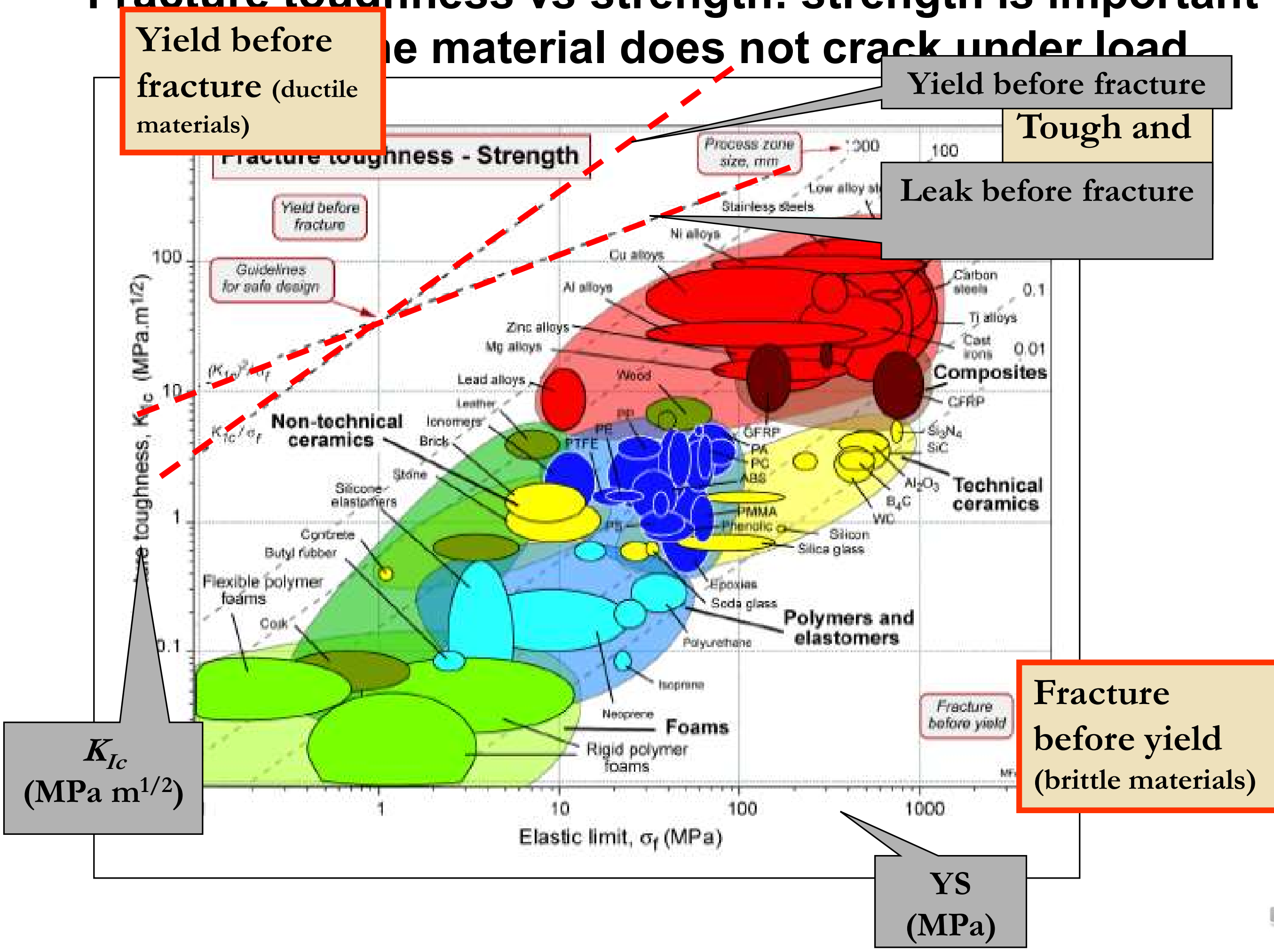
Function:
Safe Pressure Vessel

Constraints:
Radius
Wall thickness
maximum flaw size

Free Variable:
Material choice

Objective:
Maximize contained pressure

Fracture toughness vs strength: strength is important
...e material does not crack under load



Case studies in KIc- σ: Pressure vessels

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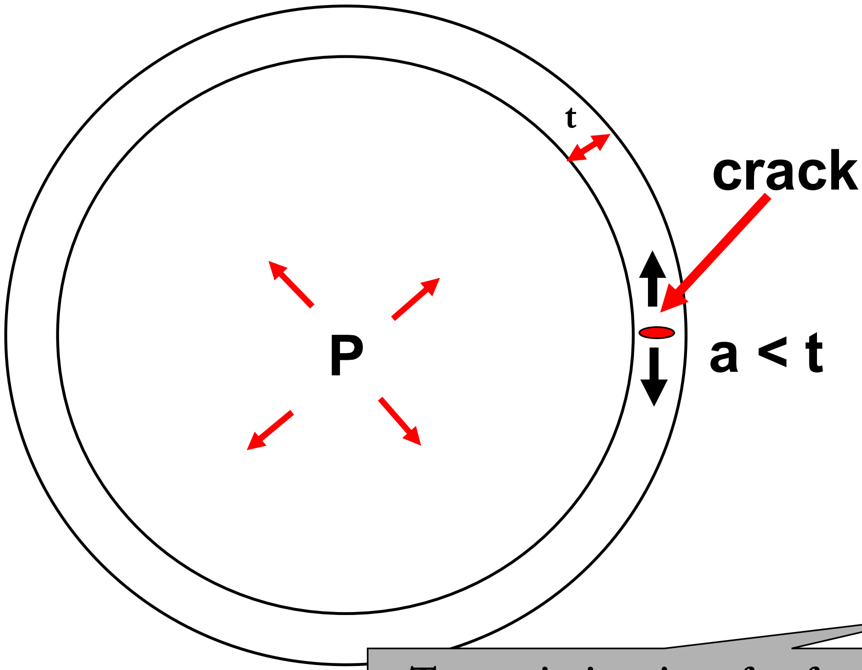
Minimize thickness	M_4
Minimize weight	M_5

Small pressure vessels: Yield before break (M3)



$$\sigma = \frac{PR}{2t} < \sigma_y$$

$$K_{IC} = \sigma^* \sqrt{\pi a}$$

$$\sigma^* = \frac{K_{Ic}}{\sqrt{\pi a}}$$

Y.B.B. => σ_y < σ^*

$$a = \frac{1}{\pi}\left[\frac{K_{Ic}}{\sigma_y}\right]^2$$

To maximise size of <u>safe</u> crack, pick materials with high K/σ_y ratio

We idealize the pressure vessel as a thin-walled sphere of radius R and wall thickness t. (The material selection aspects of the problem are independent of shape, so we choose the shape which offers the simplest analysis.) The mass of the vessel is:

$$m = 4\pi R^2 t \rho$$

In pressure vessel design, the wall thickness, t, is chosen so that, at the working pressure p, the stress is less than the yield strength, σ_y, of the wall.

$$\sigma = \frac{pR}{2t} \leq \sigma_y$$

A small pressure vessel can be examined ultrasonically, or by X-ray methods, or proof tested, to establish that it contains no crack or flaw of diameter greater than 2ac. The stress required to make such a crack propagate is

$$\sigma^* = \frac{K_{Ic}}{\sqrt{\pi a}}$$

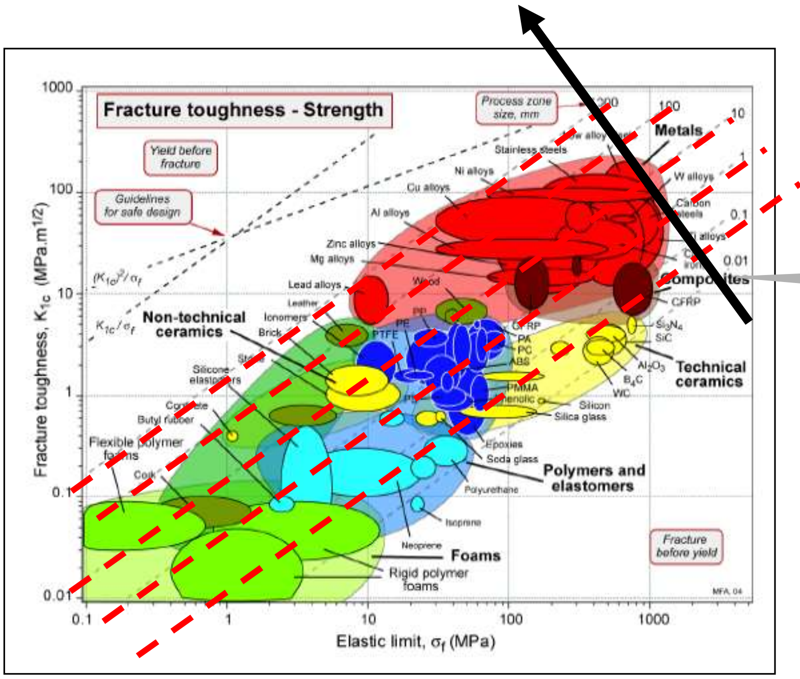
Safety obviously requires that the working stress is also less than the fracture stress of equation; but greater security is assured by requiring that the crack will not propagate even if, in an overload, the stress reaches the general yield stress. Then the vessel will deform stably in a way which can be detected. This condition is expressed by requiring that σf be greater than the yield stress, σ_y, giving

$$a = \frac{1}{\pi}\left[\frac{K_{Ic}}{\sigma_y}\right]^2$$

The tolerable crack size is maximized by choosing a material with the largest value of $M_1 = K_{IC}/\sigma_y$

Small pressure vessels: Yield before break

$$a = \frac{1}{\pi}\left[\frac{K_{Ic}}{\sigma_y}\right]^2$$



Crack size increases this way

Case studies in KIc- σ: Pressure vessels

Objectives

When designing the pressure vessel the radius will be normally fixed by design (and in this case study we will consider the radius as a constraint), however there are multiple possible objectives which we can seek when we are designing pressure vessels. In this case study we will look at 4 different objectives:

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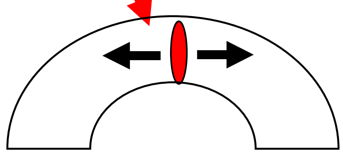
Minimize thickness	M_4
Minimize weight	M_5

Large pressure vessels: Leak before break (M2)

Set 2a = t



Maximum pressure

$$P = \frac{2t\sigma_y}{R} \quad \rightarrow \quad t = \frac{PR}{2\sigma_y}$$

$$K_{IC} = \sigma^* \sqrt{\pi a} = \sigma^* \sqrt{\pi\, t/2}$$

$$\sigma^* = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{K_{Ic}}{\sqrt{\pi\, t/2}}$$

$$\sigma^{*2} = \frac{K_{Ic}^2}{\pi\, t/2}$$

$$\sigma^{*2} = \frac{K_{Ic}^2 \sigma_y}{\pi P R}$$

$$\sigma^* = \sigma_y$$

Crack still stable at yield

$$t = \frac{PR}{2\sigma_y}$$

$$P \propto \frac{K_{Ic}^2}{\sigma_y}$$

To maximise operating pressure, pick materials with high K^2/σ_y ratio

To minimise wall thickness, maximise σ_y

Large pressure vessels cannot always be X-rayed or tested ultrasonically; and proof-testing them may be impractical. Further, cracks can grow slowly because of corrosion or cyclic loading, so that a single examination at the beginning of service life may not be sufficient. Then safety can be assured by arranging that a crack just large enough to penetrate both the inner and the outer surface of the vessel is still stable, because the leak caused by the crack can be detected. This is achieved by setting ac in equation equal to t/2:

$$\sigma^* = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{K_{Ic}}{\sqrt{\pi\, t/2}}$$

The wall thickness t of the pressure vessel was, of course, designed to contain the pressure p without yielding. This means that

$$t = \frac{PR}{2\sigma_y}$$

Substituting this into the previous equation (with σ* = σy) gives

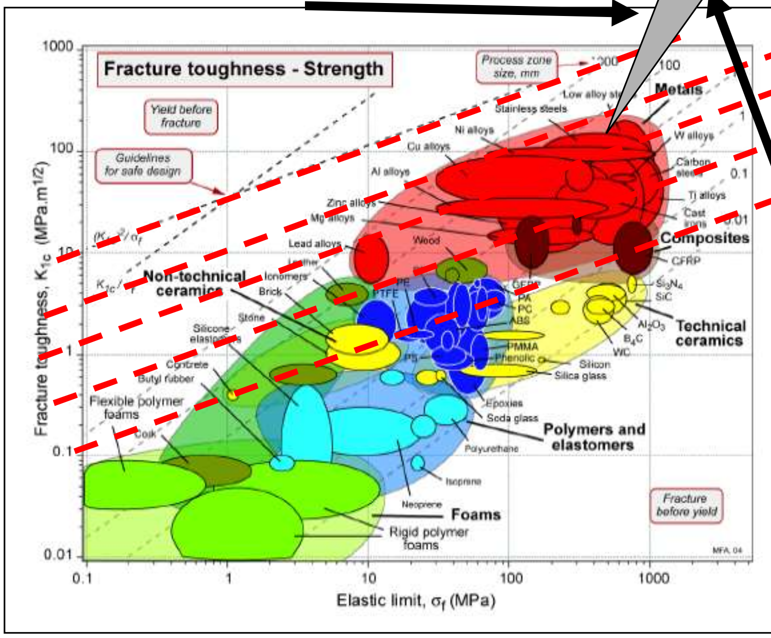
$$P \propto \frac{K_{Ic}^2}{\sigma_y}$$

Large pressure vessels: Leak before break

Wall thickness decreases this way

Pressure vessel steels

$$t = \frac{PR}{2\sigma_y}$$



$$P \propto \frac{K_{Ic}^2}{\sigma_y}$$

Operating pressure increases this way

Contour lines: Case studies in K_{Ic}-E

Design requirements

Function	*What does the component do ?*	***A tension tie***
Constraints	*What essential conditions must it meet ?*	***A crack of a given length***
Objectives	*What measure of performance is to be maximized or minimized ?*	***maximise load*** ***maximaise displacement*** ***maximise absorbed energy***
Free variables	*Which design variables are free ?*	***Choice of material***

Case studies in KIc- σ: Pressure vessels

Objectives

When designing the pressure vessel the radius will be normally fixed by design (and in this case study we will consider the radius as a constraint), however there are multiple possible objectives which we can seek when we are designing pressure vessels. In this case study we will look at 4 different objectives:

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Minimize thickness	M_4
Minimize weight	M_5

ALL IN EXERCISE!

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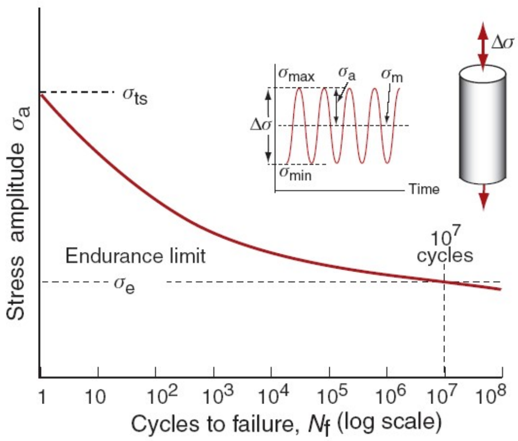
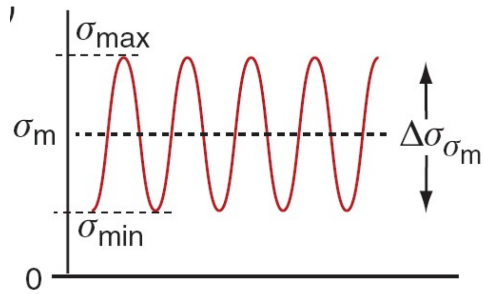
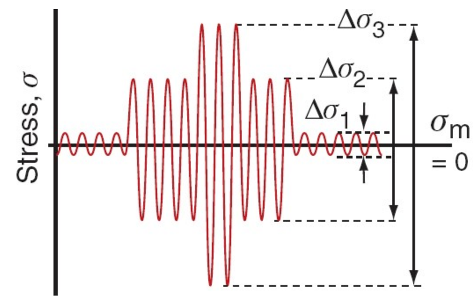
High cycle fatigue



Wöhler curve, S-N curve: endurance limit



Goodman's law:
accounts for finite mean stress

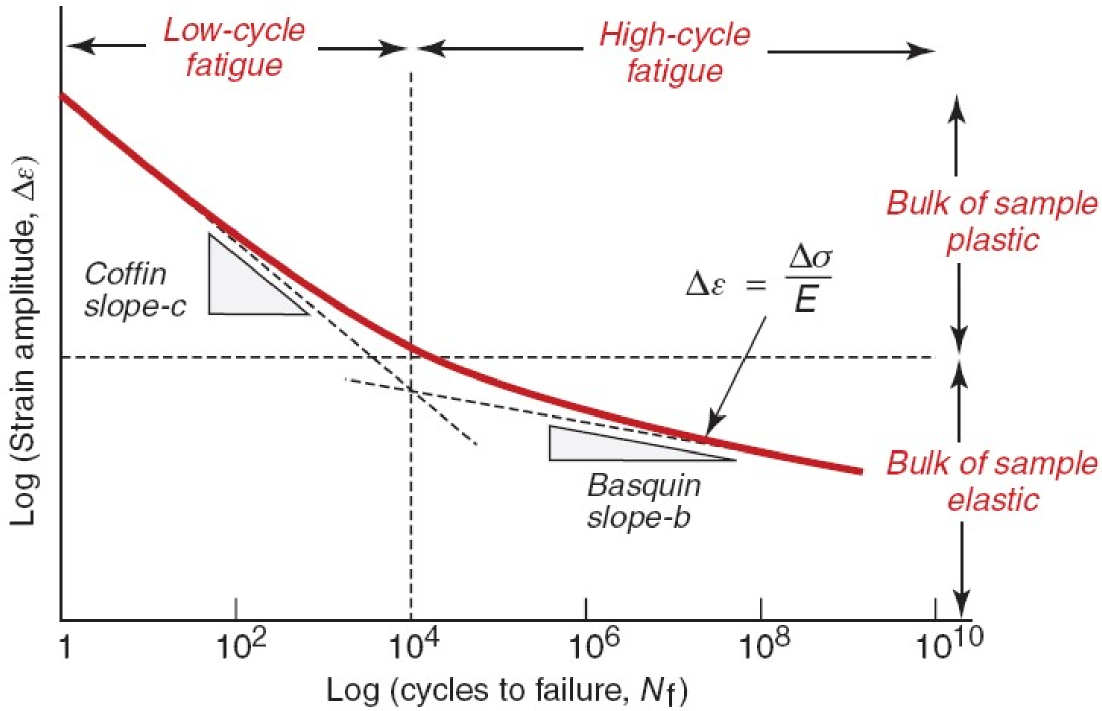


Miner's law:
Accounts for variable amplitude

High cycle fatigue testing was first carried out by a German engineer A. Wöhler.

Stress amplitude is plotted against the log of the number of cycle to failure. The endurance limit is the stress amplitude below which fracture does not occur at all, or occurs after a very large number of cycles (10^7).

Material properties: fatigue

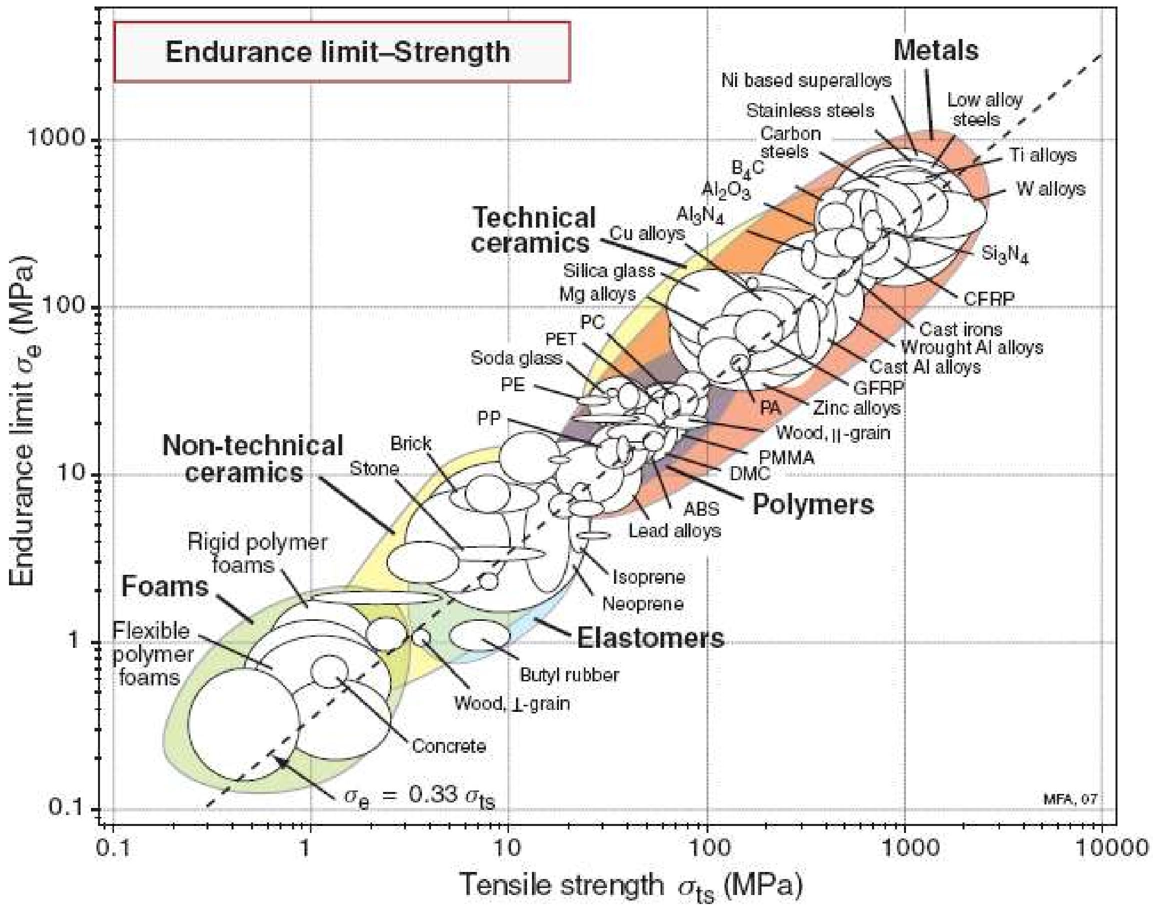


Fatigue failure is insidious: little evidence until sudden failure!

Low cycle fatigue: peak stress exceeds yield; Coffin slope c ~ 0.5

High cycle fatigue: peak stress much below yield, elastic loading; Basquin slope b ~ 0.07

Endurance limit



The endurance limit of metals and polymers cluster around the line; $\sigma_e \approx 0.33\ \sigma_{ts}$

For ceramics and glasses; $\sigma_e \approx 0.9\ \sigma_{ts}$

As the endurance limit is obtained from simple laboratory tests, several correction factors are used to account for real life situations

ka = surface finish factor (machined parts have different finish)

kb = size factor (larger parts greater probability of finding defects)

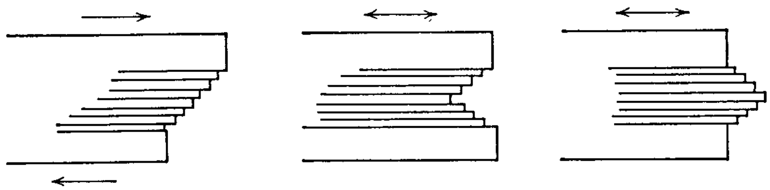
kc = reliability / statistical scatter factor (accounts for random variation)

kd = operating T factor (accounts for diff. in working T & room T)

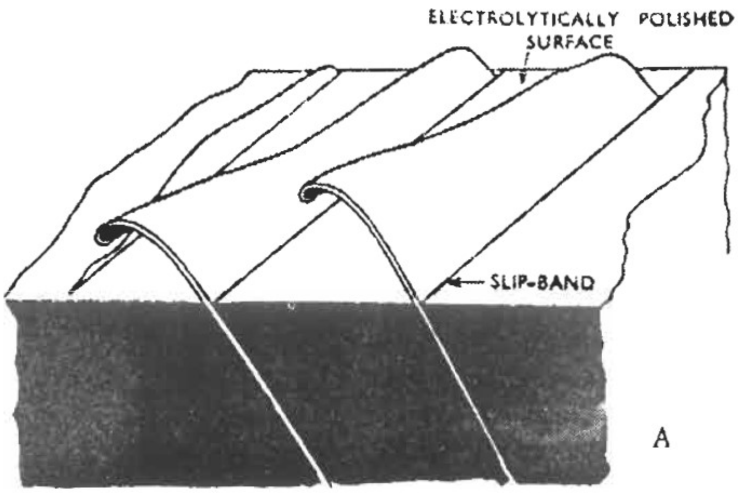
ke = loading factor (differences in loading types)

……..

Origins of fatigue: Crack initiation



Wood‘s model (1959):
Intrusions & extrusions

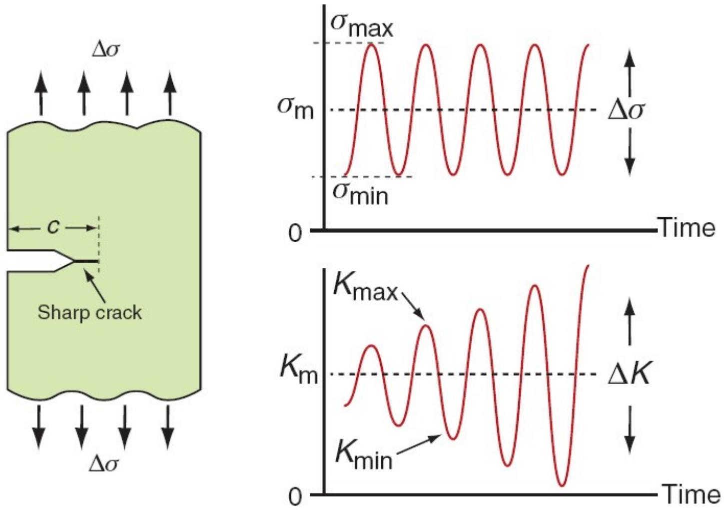


Wood’s model suggested how strain accumulation by slip can lead to fatigue crack initiation.

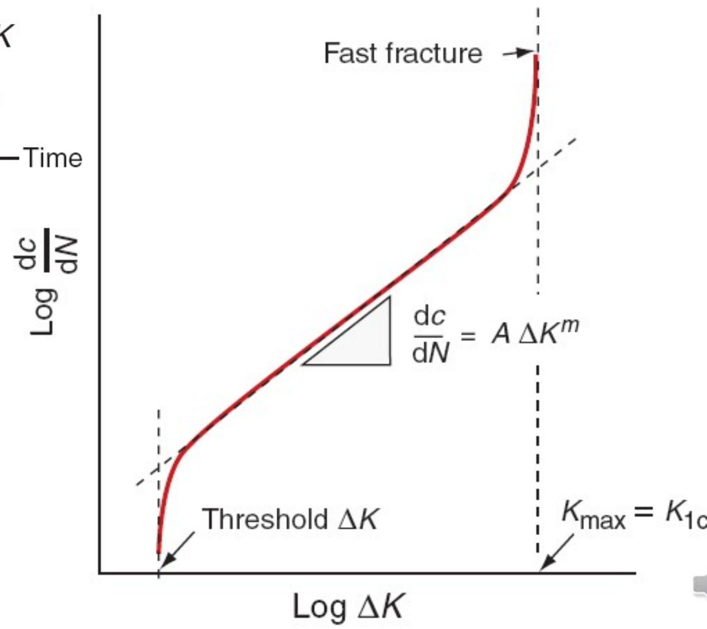
The temporary persistence of the slip gave rise to the term “Persistent Lüders Bunds” (PLBs) to describe the regions of currently active slip.

Bands which were previously active and had become quiescent can be reactivated if the other volumes of the specimen have undergone slipping, temporary persistence, gradual hardening, and subsequent quiescence.

Fatigue loading of cracked components: Paris law



Characterization of
fatigue crack propagation:
Fracture mechanics approach



Paris‘ law: m~2-4 (metals)

$$\frac{dc}{dN} = A\left(\Delta K\right)^m$$

Importance of the Fracture mechanics approach:

1. Cracks are inevitable! NDT methods tell us that there are no cracks longer than the resolution limit of the technique.
2. K_{Ic} can be evaluated.

Fatigue life prediction: Living with cracks!

$$\frac{dc}{dN} = A(\Delta K)^m \qquad \Delta K = K_{max} - K_{min} = \Delta\sigma\sqrt{\pi c}$$

$$\frac{dc}{dN} = A(\Delta\sigma)^m (\pi c)^{m/2}$$

$$N_t = \frac{1}{A*(\Delta\sigma)^m}\int_{c_i}^{c^*}\left(\frac{dc}{(\pi c)^{m/2}}\right)$$

c^* - critical crack length; value at which fast fracture will occur
c_i – initial crack length

Calculation of life-limited fatigue crack growth:

Casings of Steam turbines, chemical enginnering equipment, boilers and pipe-work are assumed to contain cracks during service.

Assume a 10 mm long crack in a steel tank with m = 4, A = 2.5 × 10^{-6}

If the strength and toughness of the steel tank are σ_o = 90 MPa and K_{Ic}=45 MPa.$m^{1/2}$ respectively, how long can it be used safely provided the stress amplitude does not exceed 1.5 MPa?

How can does one prevent a catastrophic failure?

Solution:

c_i = 10 mm, c* = 80 mm and $\Delta\sigma$ = 1.5 MPa

N_t = 7 × 10^6

Leak before break criterion: The thickness is less than c*.

Fatigue life calculation: Example

Assume a 10 mm long crack in a steel tank with m = 4, A = 2.5 × 10^{-6}

If the strength and toughness of the steel tank are σ_o = 90 MPa and K_{Ic}=45 MPa.$m^{1/2}$ respectively,

a) How long can it be used safely provided the stress amplitude does not exceed 1.5 MPa?

b) How can one prevent a catastrophic failure?

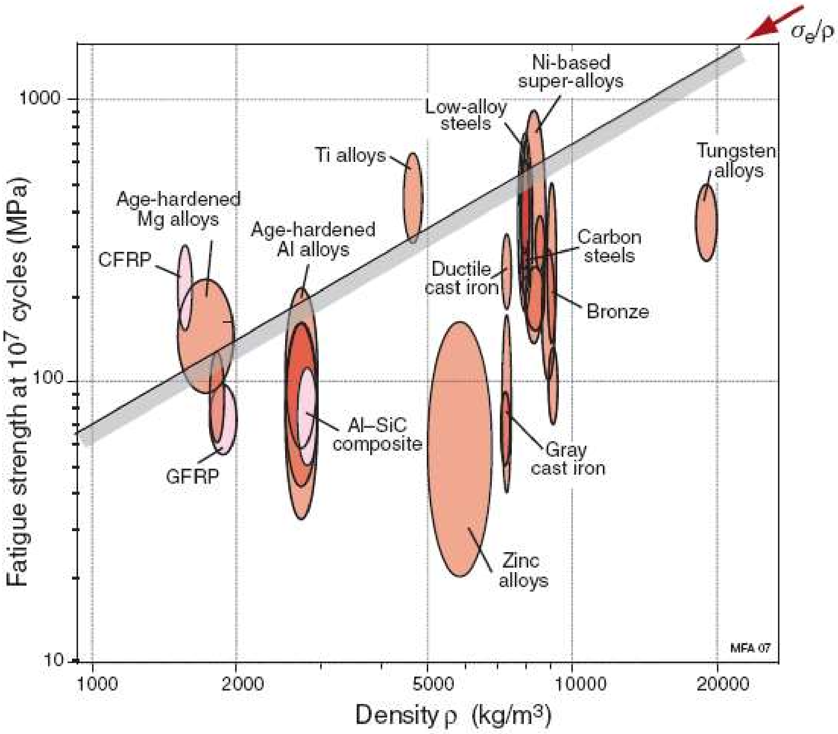
Solution:

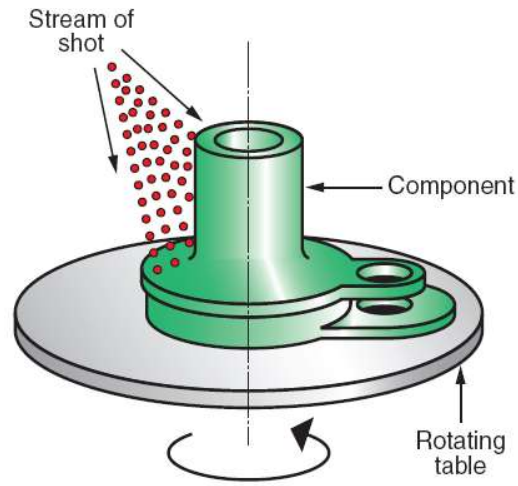
c_i = 10 mm, c* = 80 mm (from K_{Ic}=45 MPa.$m^{1/2}$) and $\Delta\sigma$ = 1.5 MPa

a) N_t = 7 × 10^6

b) Leak before break criterion: The thickness should be less than c*.

Manipulating resistance to fatigue





Materials with high ratios of σ_e / ρ are desirable: Ti-alloys and CFRP!

Shot-peening: Compressive residual stress on the surface enhances fatigue life of components

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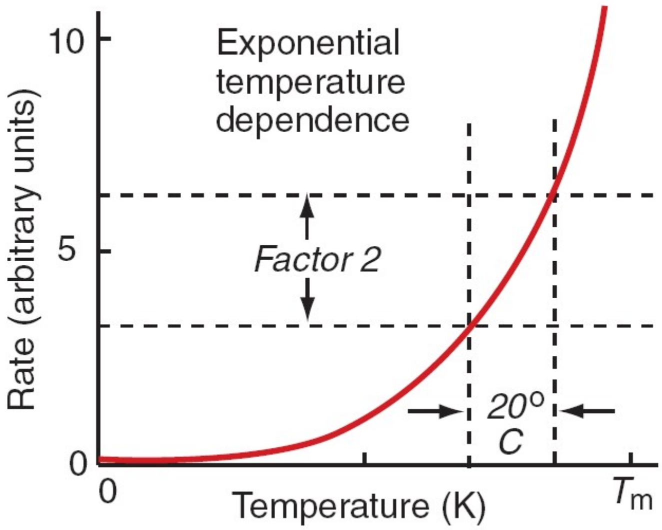
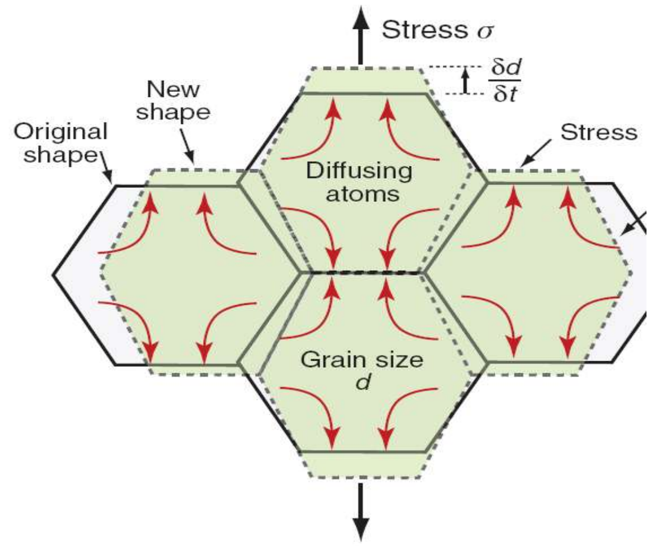
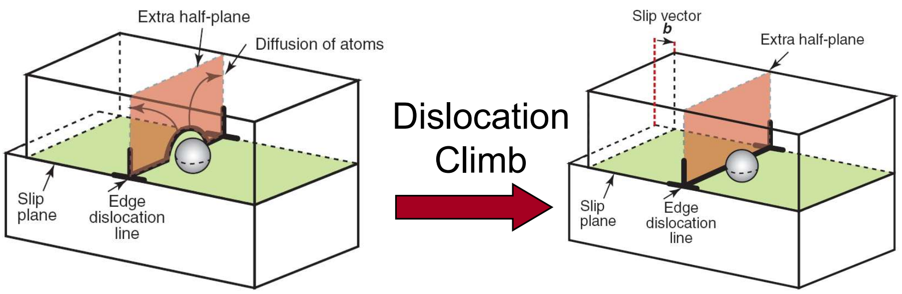
Creep mechanisms





Diffusional flow



Power-law creep

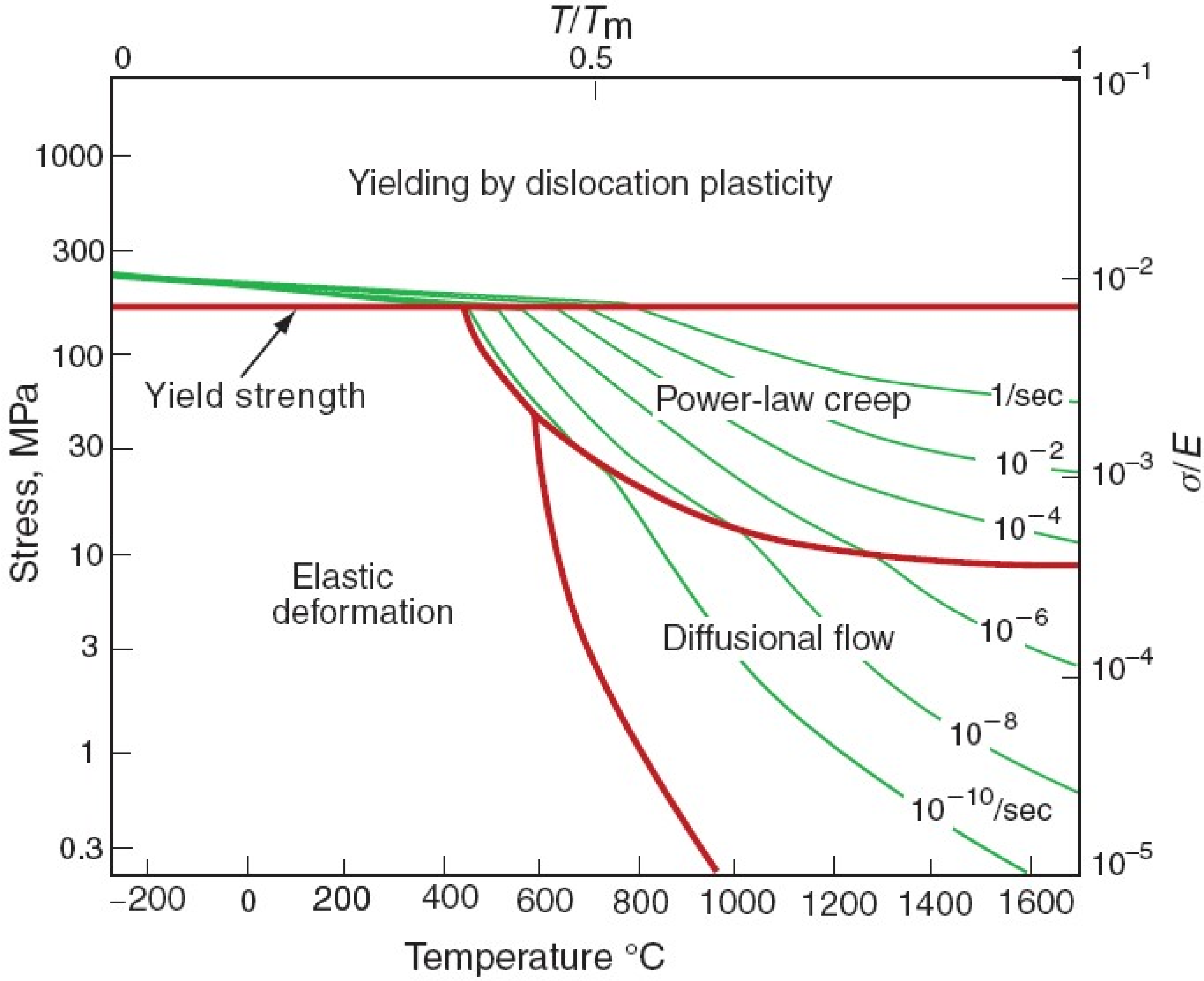
$$\dot{\varepsilon}_{ss} = A \times \sigma^{n} \times \exp - \left(\frac{Q}{RT} \right)$$

Creep is a diffusion-dominated process: exponential dependence on temperature!

Diffusion unlocks dislocations from obstacles in their path: Dislocation climb; occuring and measurable only at T~0.35Tm

The exponential dependence on temperature together with the power law dependence on stress: Power-law creep

Deformation Mechanism Map



Deformation Mechanism Map:

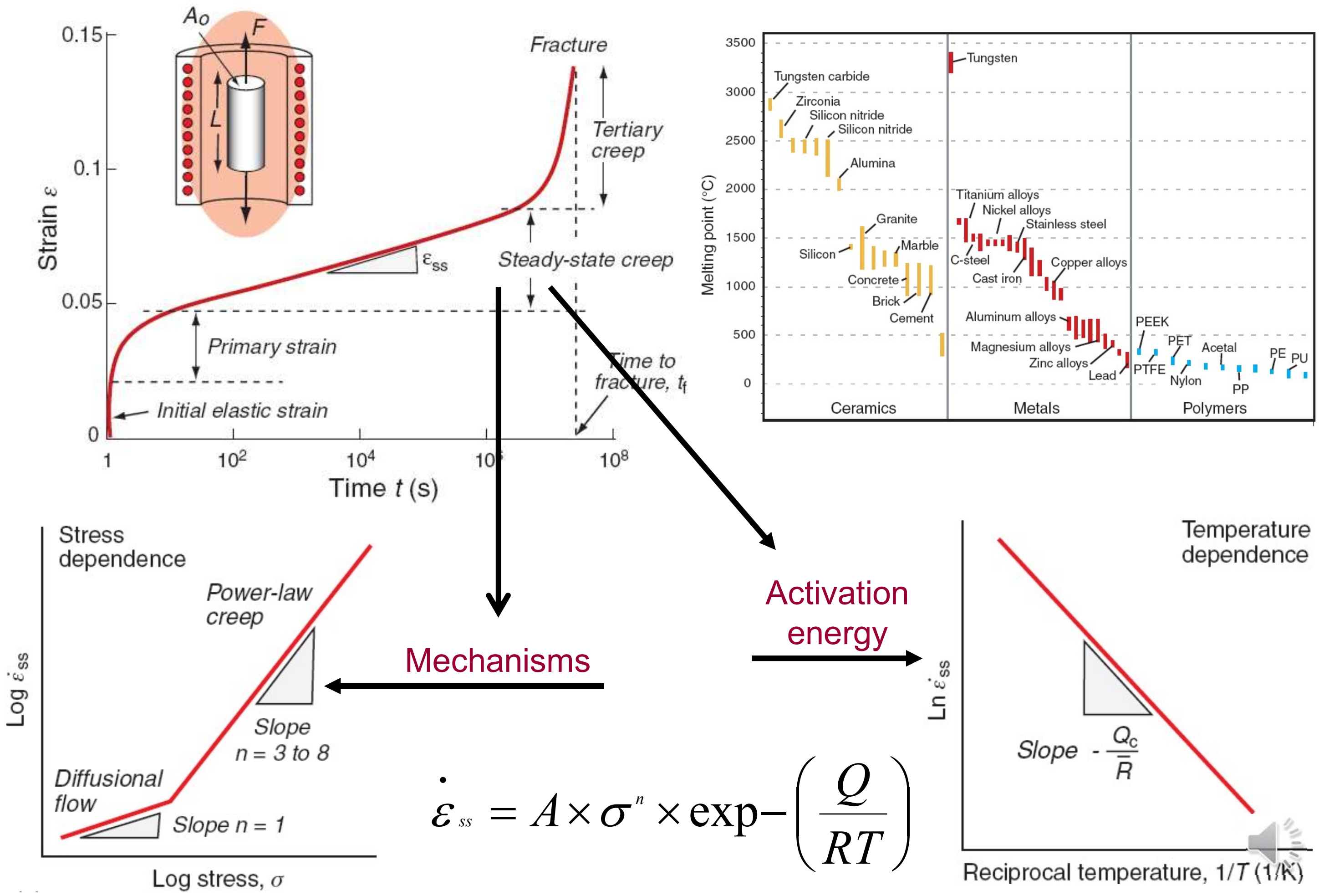
A summary of competing deformation mechanisms in a material; stress, temperature and strain-rate combinations

It is helpful in selecting materials for high temperature application

Red lines in the map delimit domains of dominating deformation mechanisms

Green lines are overlapping strain rate contours

High temperature materials: Creep



$$\dot{\varepsilon}_{ss} = A \times \sigma^{n} \times \exp - \left(\frac{Q}{RT} \right)$$

Creep: Plastic deformation in materials at T > 0.5 T_m (Can Ice creep?)

Classified into 3 stages as a function of time:

Stage I: Primary creep; decreasing strain rate regime

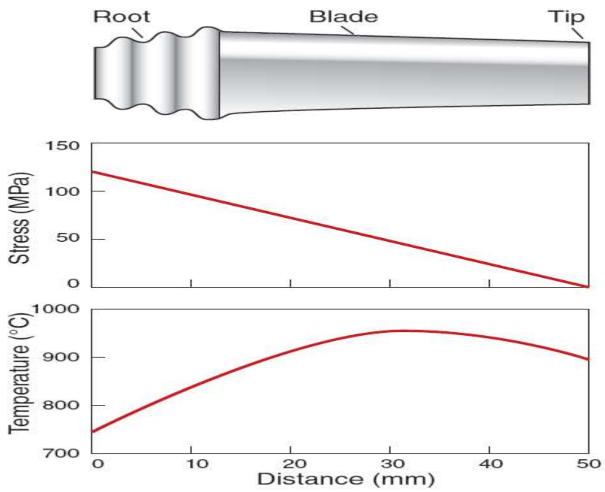
Stage II: Secondary creep; constant strain rate or steady state regime

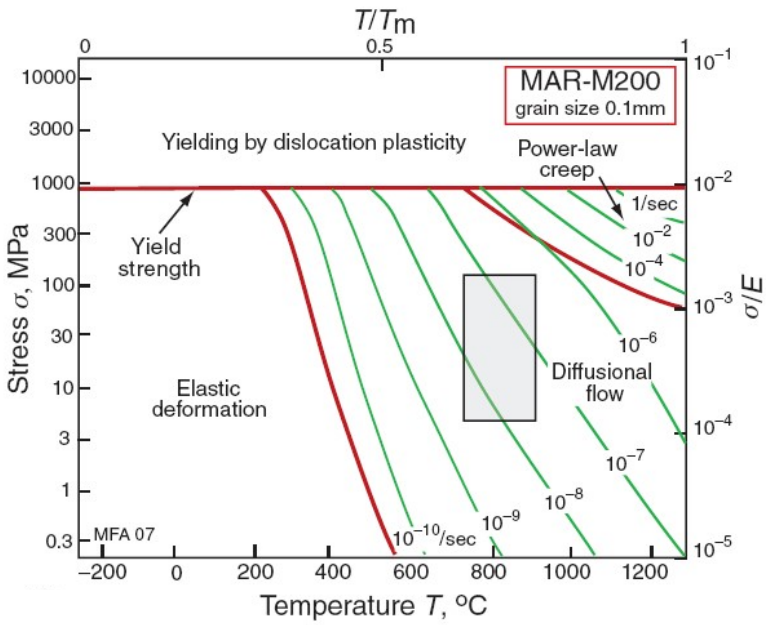
Stage III: Primary creep; increasing strain rate regime leading to fracture

Typically the log of the steady state creep rate is plotted against:

1. The log of the stress to distinguish between the operating creep mechanisms.
2. The reciprocal of the absolute temperature to obtain the activation energy of the creep process.

Coping with creep!





MAR-M200: typical Ni-based super-alloy used as turbine blades (single crystal) because of high-temperature strength, toughness and oxidation resistance.

Note the stress and temperature profiles from the root to the tip.

Tolerances are strict: Creep strain during service should be minimal, otherwise the blade might touch the casing during operation leading to catastrophic failure.

The strengthening mechanisms are such that only diffusional flow contributes to creep.

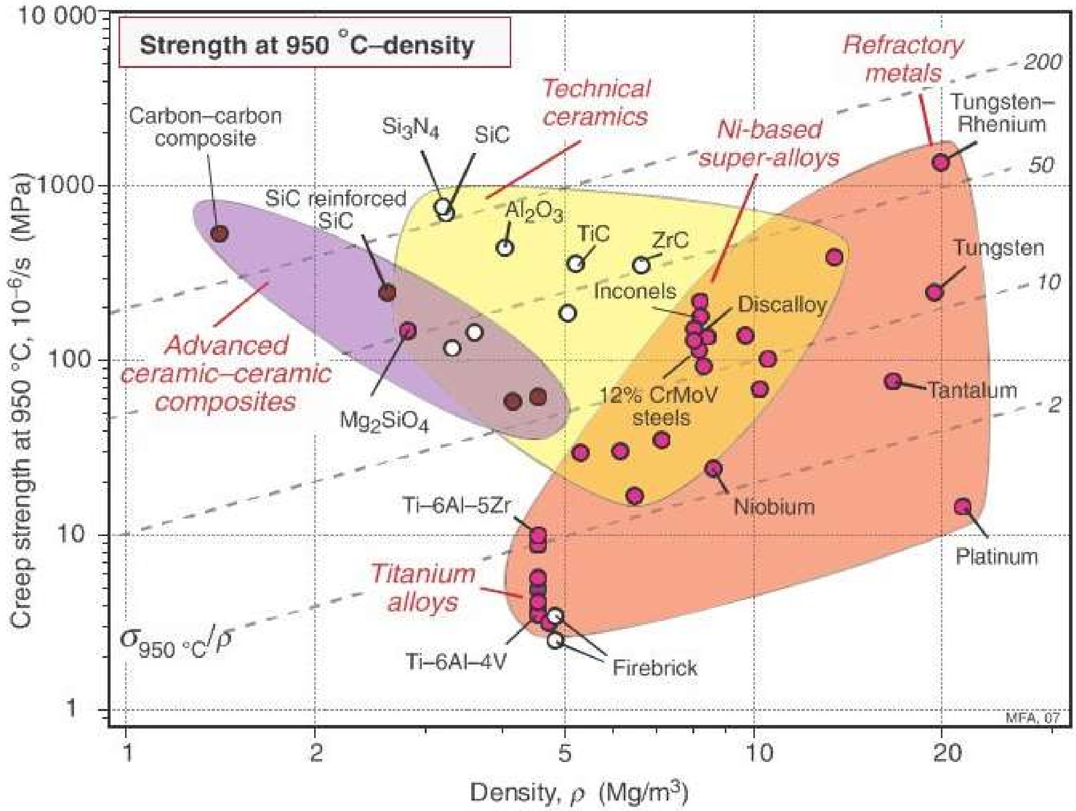
This can be further reduced by using a single crystal.

Coping with creep!

A stainless steel tie of length $L = 100$ mm is loaded in tension under a stress $\sigma = 150$ MPa at a temperature $T = 800$ °C. The creep constants for stainless steel are: reference strain-rate $\dot{\varepsilon}_o = 10^6/s$, reference stress $\sigma_o = 100$ MPa, stress exponent $n = 7.5$ and activation energy for power-law creep $Q_c = 280$ kJ/mol. What is the creep rate of the tie? By how much will it extend in 100 hours?

Answer. The steady state creep rate is given by $\dot{\varepsilon}_{ss} = \dot{\varepsilon}_o \left(\frac{\sigma}{\sigma_o} \right)^n \exp - \left(\frac{Q_c}{\bar{R}T} \right)$. Using the data listed above, converting the temperature to Kelvin, and using the gas constant $\bar{R} = 8.31$ J/mol · K gives $\dot{\varepsilon}_{ss} = 4.8 \times 10^{-7}/s$. After 100 hours ($3.6 \times 10^5$ seconds) the strain is $\varepsilon = 0.173$ and the 100 mm long tie has extended by $\Delta L = 17.3$ mm.

Creep strength diagram



A chart showing the strength of selected material at a particular high temperature (950°C) and a strain rate of 10-6/sec – plotted against density.

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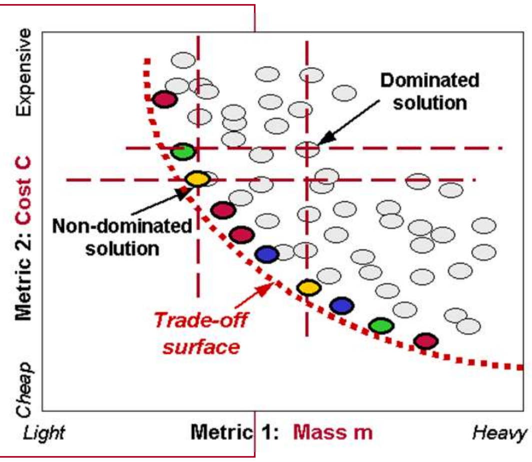
Objectives and Constraints in conflict:

trade-off methods and penalty functions



Outline

- Conflicting constraints, conflicting objectives
- Multi-objective optimisation
- Trade-off methods
- Penalty functions and exchange constants
- *Exercise*

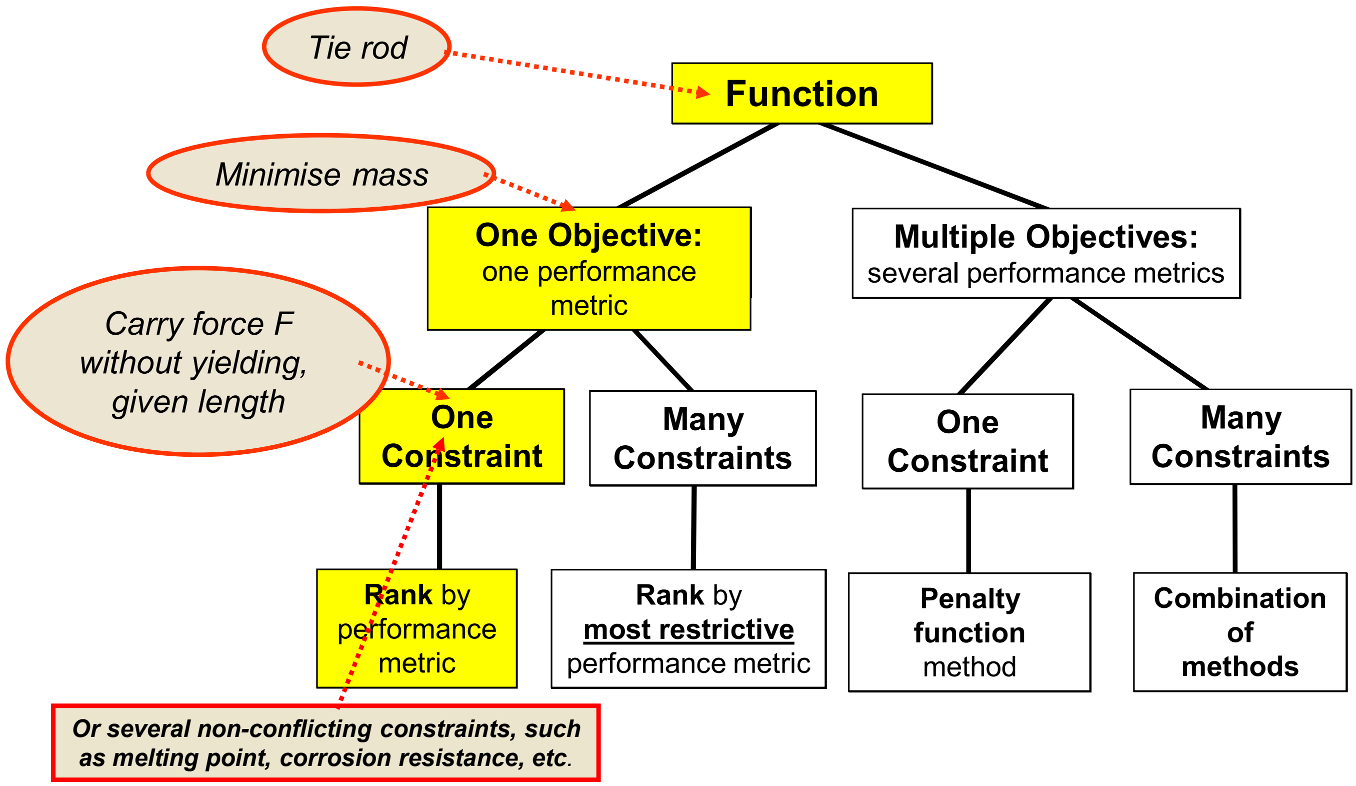
More info:
- "Materials Selection in Mechanical Design", **Chapters 9 and 10**

Real-life decision-making frequently requires that a compromise be reached between **conflicting objectives or conflicting constraints**. Some are only too familiar: the compromises required to strike a balance between the performance and the cost of a car for example, or between health and the pleasure of eating rich foods, or between wealth and quality of life. Conflict arises because the choice that optimizes one objective will not, in general, do the same for the others; then the best choice is a compromise, optimizing none but pushing all as close to their optima as their interdependence allows.

Multiple Constraints and Objectives

Simplest case:

Design with one objective, meeting a single constraint



Multiple constraints and objectives

Design requirements impose **constraints** on material choice
And identify **objectives** - criteria for optimising the choice

Typical constraints

The material must be

- Electrically conducting
- Optically transparent.....

And meet target values of

- Stiffness
- Strength…..

And be able to be

- Die cast
- Welded ......

Typical objectives

Minimize

- Mass *(satellite components)*
- Volume *(mobile phones)*
- Energy consumption *(fridges)*
- Carbon footprint *(cars)*
- Cost (*everything*)

Dealing with multiple constraints is straightforward

Dealing with multiple objectives needs **trade-off methods**

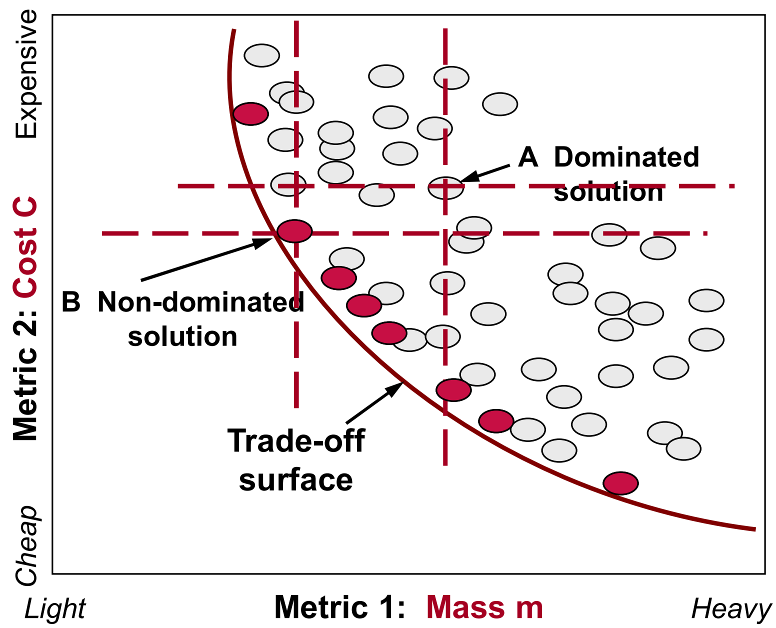
Take, as example, simultaneously minimizing **mass m** and **cost C**

This frame lists, on the left, typical constraints that a material must meet. Dealing with **multiple constraints** is straightforward – just apply them using Limit, Graph and Tree stages. On the right is a list of typical objectives. Dealing with multiple objectives is more complicated.

An **objective**, it will be remembered from Units 3 and 4, defines a performance metric. If the objective is to *minimize mass*, then the mass becomes the metric of "goodness" or "badness" of a given choice: the lightest solution that meets all the constraints of the problem is the best choice. If the objective is to *minimize cost*, then the cheapest solution that meets all constraints is the best choice. The metric allows solutions to be ranked. This frame lists common design objectives; there are, of course, many more. It is rare that a design has only one objective. And when there are two a conflict arises: the choice that minimizes one metric – mass say – does not generally minimize the other – cost, for example. Then a compromise must be sought. To reach it we need some simple ideas drawn from the field of **multi-objective optimization.**

Multi-objective optimisation: the terminology

- ***Solution:*** a viable choice, meeting constraints, but not necessarily optimum by either criterion.
- ***Plot solutions*** as function of performance metrics. (Convention: express objectives to be minimized)
- ***Dominated solution:*** one that is unambiguously non-optimal (as **A**)
- ***Non-dominated solution:*** one that is optimal by one metric (as **B:** optimal by one criterion but not necessarily by both)



- ***Trade-off surface:*** the surface on which the non-dominated solutions lie (also called the Pareto Front)

Multi-objective optimization is a technique for reaching a compromise between conflicting objectives. It lends itself to visual presentation in a way that fits well with methods developed here thus far. This frame explains the words. They are illustrated by the diagram on the right in which we have specialized a problem to a trade-off between the mass of a component and its cost.

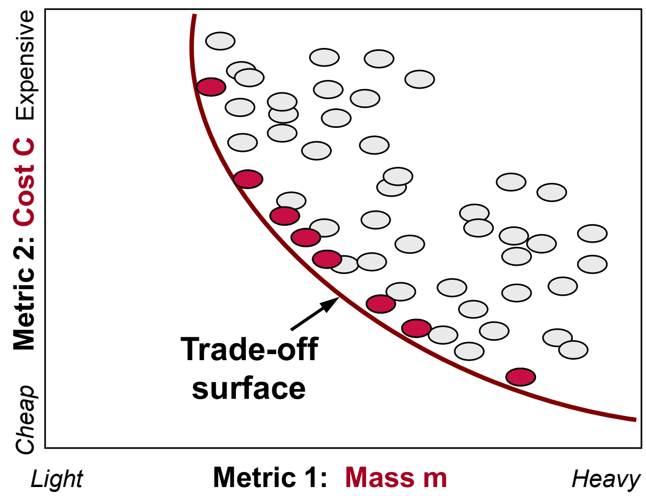
The first bullet point on the frame defines a *solution*: a choice of material to make a component that meets all the necessary constraints and is thus a candidate for the design, although not perhaps the best one. The little circles each represent a solution; each describes the mass and cost of the component if made from a given material. The next two bullet points distinguish between a *dominated* solution (meaning that other solutions exist that are both lighter and cheaper) and a *non-dominated* solution (one that is lighter than all others that cost less and cheaper than all others that are lighter – thus there is no other solution that is both lighter and cheaper than it is). The lower envelope links non-dominated solution. It defines the **trade-off surface** or **Pareto front.** Solutions that lie on or near the trade-off surface are a better choice than those that do not.

We adopt the convention that each performance metric is defined in such a way that a minimum is sought for it. For mass and cost, that is exactly what we want. But if the metric were maximum speed v (a performance objective for a sports car, for instance) we must invert it and seek a minimum for 1/v. With this convention the trade-off surface must have a negative slope everywhere, as that in the schematic does. A positive slope would link non-dominated solutions.

With this background we can examine strategies for finding the best compromise. There are three.

Finding a compromise: strategy 1

- **Make trade-off plot**
- **Sketch trade-off surface**
- **Use intuition** to select a solution on the trade-off surface

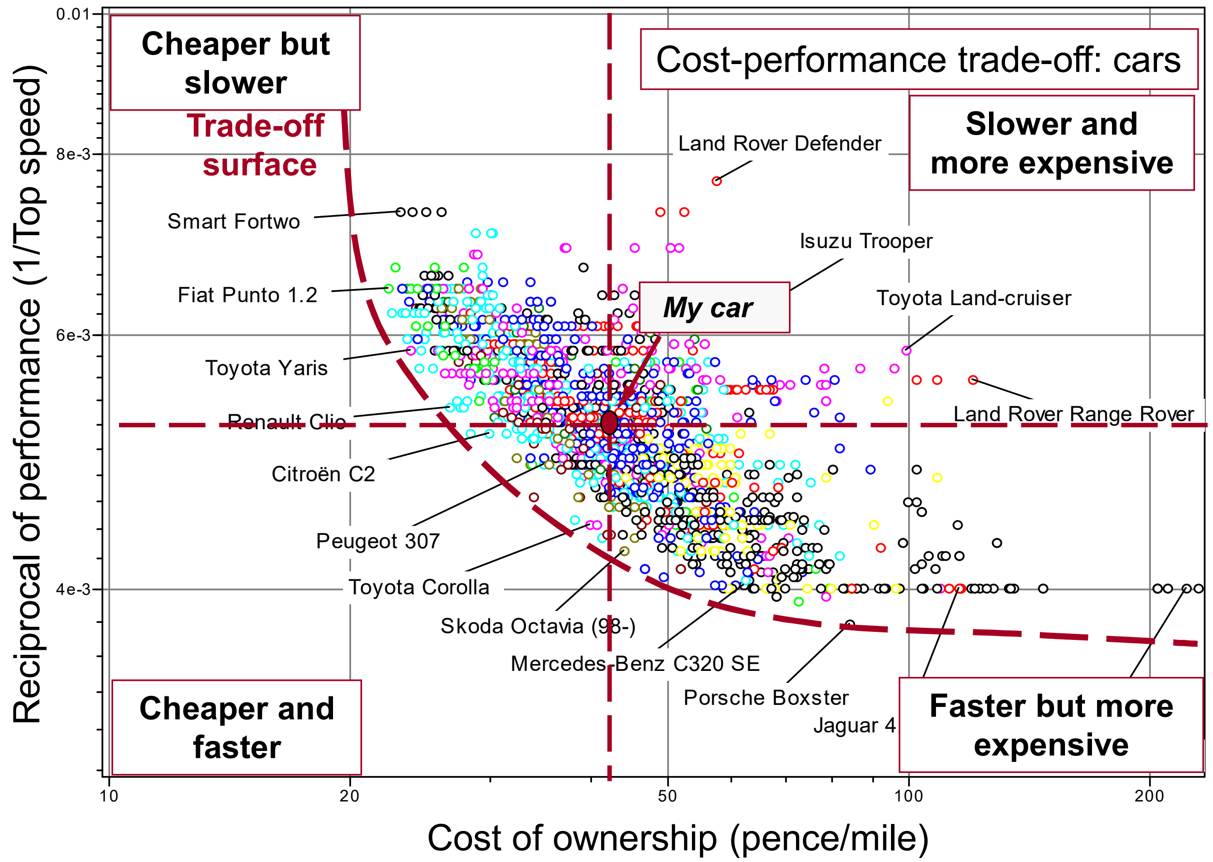


- "Solutions" on or near the surface offer the best **compromise** between mass and cost

- Choose from among these; the choice depends on how highly you value a light weight, -- a question of ***relative values***

The solutions on or near the trade-off surface offer a better compromise between mass and cost than those that do not. This immediately isolates a subset of the entire population of solutions, identifying these as the best candidates. It is a big step forward, but it still leaves us with a choice: which part of the trade-off surface is the best? The first strategy is to use **intuition** (experience, good judgement, common sense – call it what you like) for guidance.

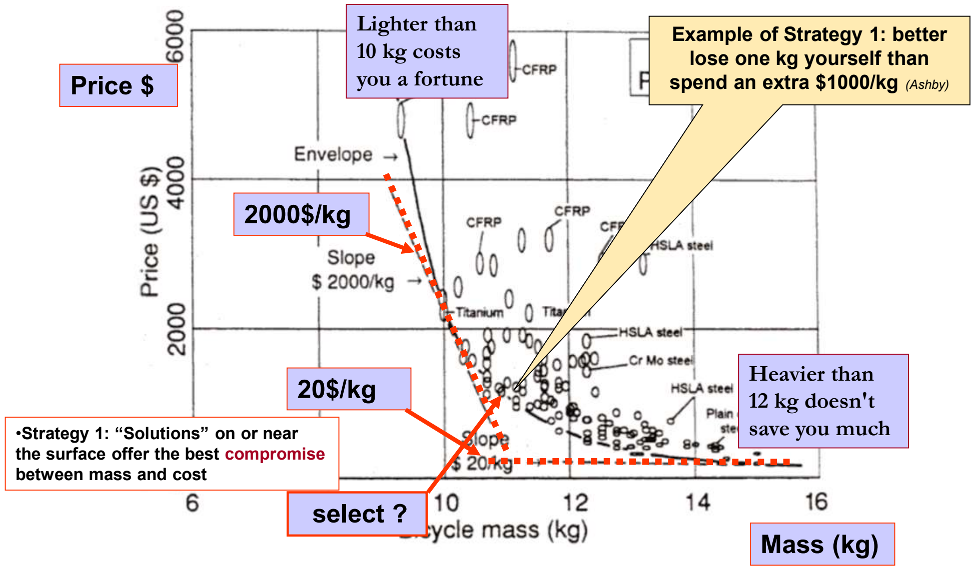
Cars: Cost-Performance trade off



Here is a **two-objective problem** in choosing a car. The aim is to select a model such as to maximize the top speed but at the same time minimize the cost of ownership. The chart show data for some 5000 models. The X-axis is the cost of ownership here measured in units of pence/mile. The Y-axis is the reciprocal of the top speed, 1/v (reciprocal because we must express the objective as a quantity to be minimized).

The data show a well defined lower envelope, the trade-off surface. Models on or near the trade-off surface offer the best compromise.

Example of Strategy 1: Price *vs*. mass of pushbikes



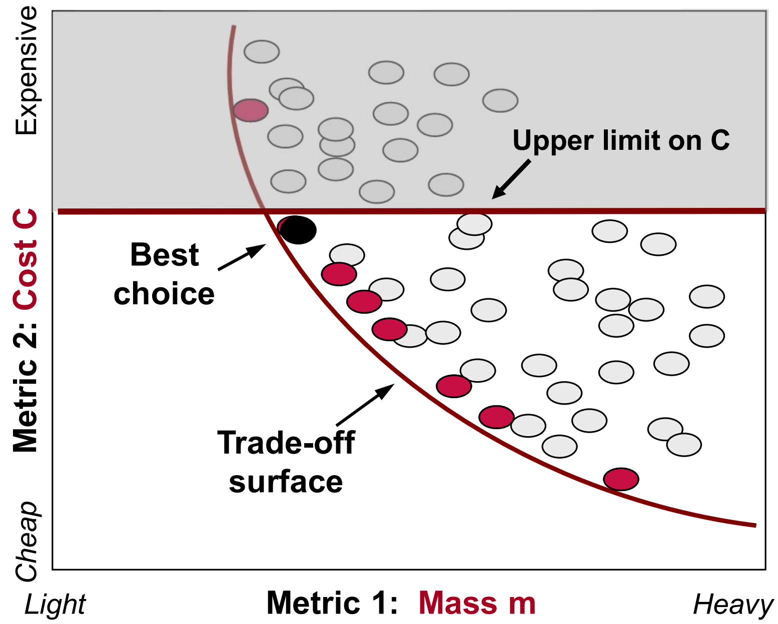
Finding a compromise: strategy 2

• ***Reformulate*** all but one of the objectives as constraints, setting an an upper limit for it

Good if budget limit

• Trade-off surface gives the best choice within budget

• BUT….not true optimisation; cost is treated as *constraint*, not *objective*.



The second tactic is to **impose an upper limit on one of the metrics** – cost, say – allowing any choice that is less than this limit. Then it's easy. Choose the solution on the trade-off surface that comes just under the limit. If you were choosing a car and wanted the fastest but had a definite budget limit, then this is the way to do it. But it is an extreme sort of optimization: cost has been treated as a constraint, not an objective. Strategies 1 and 2 help with all trade-off problems in material selection, but they rely to some extent on judgement. A more systematic method is possible – it comes next.

Finding a compromise: strategy 3

Define locally-linear
Penalty function Z

$$Z = \alpha m + C$$

Seek material with smallest Z:

- Either **evaluate Z** for each solution, and rank,

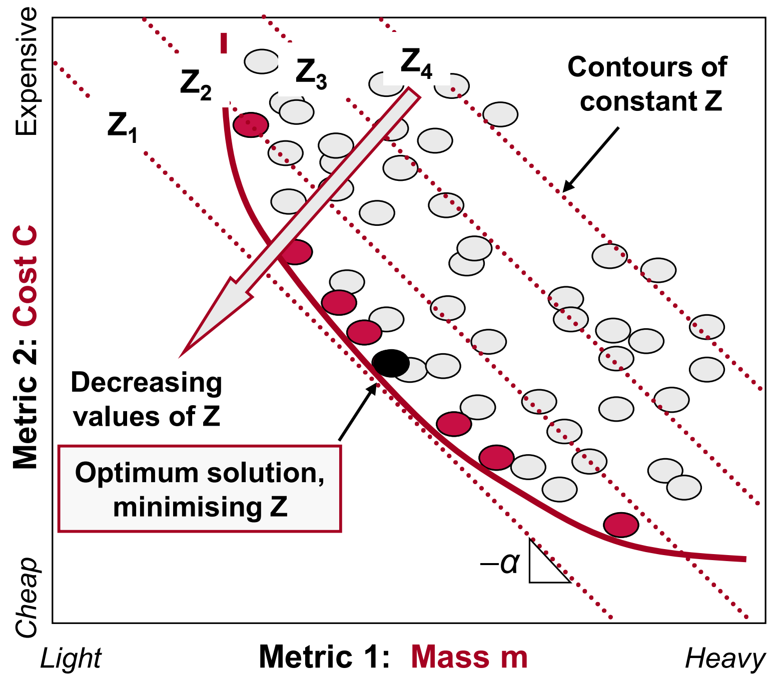
Or make **trade-off plot**

- plot on it contours of Z

$$C = -\alpha m + Z$$

-- lines of constant Z have slope -α

- **Read off** solution with lowest Z



But what is the meaning of α ?

There is a more formal, systematic, way to find the best compromise, although it is not always practical to use it. We define a **locally-linear penalty function** (a global objective) combining the two metrics mass, m, and cost, C:

$$Z = \alpha m + C$$

and seek the solution that minimizes Z (assuming we have a value for the constant α). That can be done by simply calculating Z for each solution and ranking the solutions by this value, or it can be done graphically in the way shown on this frame. Rearranging the equation for Z gives

$$C = -\alpha m + Z$$

This equation describes a family of parallel lines with slope -α, each line corresponding to a value of Z, as shown. The best choices lie near the point at which one of these lines is tangent to the trade-off surface, since this minimizes Z.

The exchange constant α

$$Z = \alpha m + C$$

The quantity α is called an "exchange constant" -- a measure of the **value of performance**, here: **value of saving 1 kg of mass.**

Exchange constants for mass saving

Transport system	α ($ per kg)
Family car	0.5 to 1.5
Truck	5 to 20
Civil aircraft	100 to 500
Military hardware	500 to 2000
Space vehicle	3000 to 10,000

How get values of α?

- Full life costing: fuel saving, extra payload
- Analysis of historic data;
- Interviews with informed planners

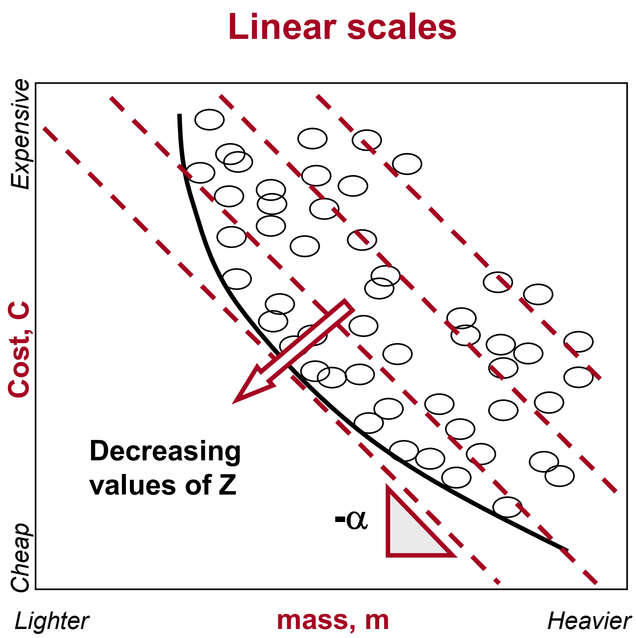
The quantity α is called an **exchange constant** (or "**parameter influence coefficient**") because it converts the units of one metric – mass – into the other – cost (like the currency exchange rate that converts one currency into another). It measures the value of a unit change of the performance metric m: it is the value associated with unit reduction in mass, and so has the units £/kg or $/kg. The table lists approximate values for α for transport systems, based on the economic benefit of a reduction in structural mass of 1kg, all other things remaining the same. For the family car it is calculated from the fuel saving over a life of 100,000 km. For the truck, aircraft and spacecraft it is calculated from the value of an additional 1kg of payload over the operating life. The values vary widely. The value of weight saving in a car is small; that is one reason that it is difficult to replace steel with a lighter metal in cars – the weight (and thus fuel) saving does not compensate for the higher cost of the material. But in space it is different: here, because launch costs per kg are so enormous, the saving of mass is valued highly, making it economic to use even very expensive materials if they save weight.

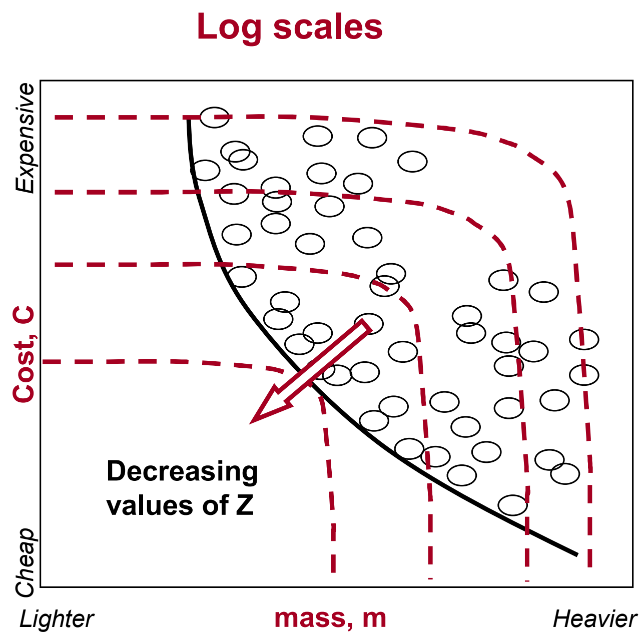
These values for exchange constants are based on engineering criteria. Sometimes, however, value is set in other ways. The *perceived value* of a product is an important factor in marketing. It is measured – or estimated – by market surveys, questionnaires and the like.

Penalty function on log scales

- A **linear** relation, on **log** scales, plots as a curve

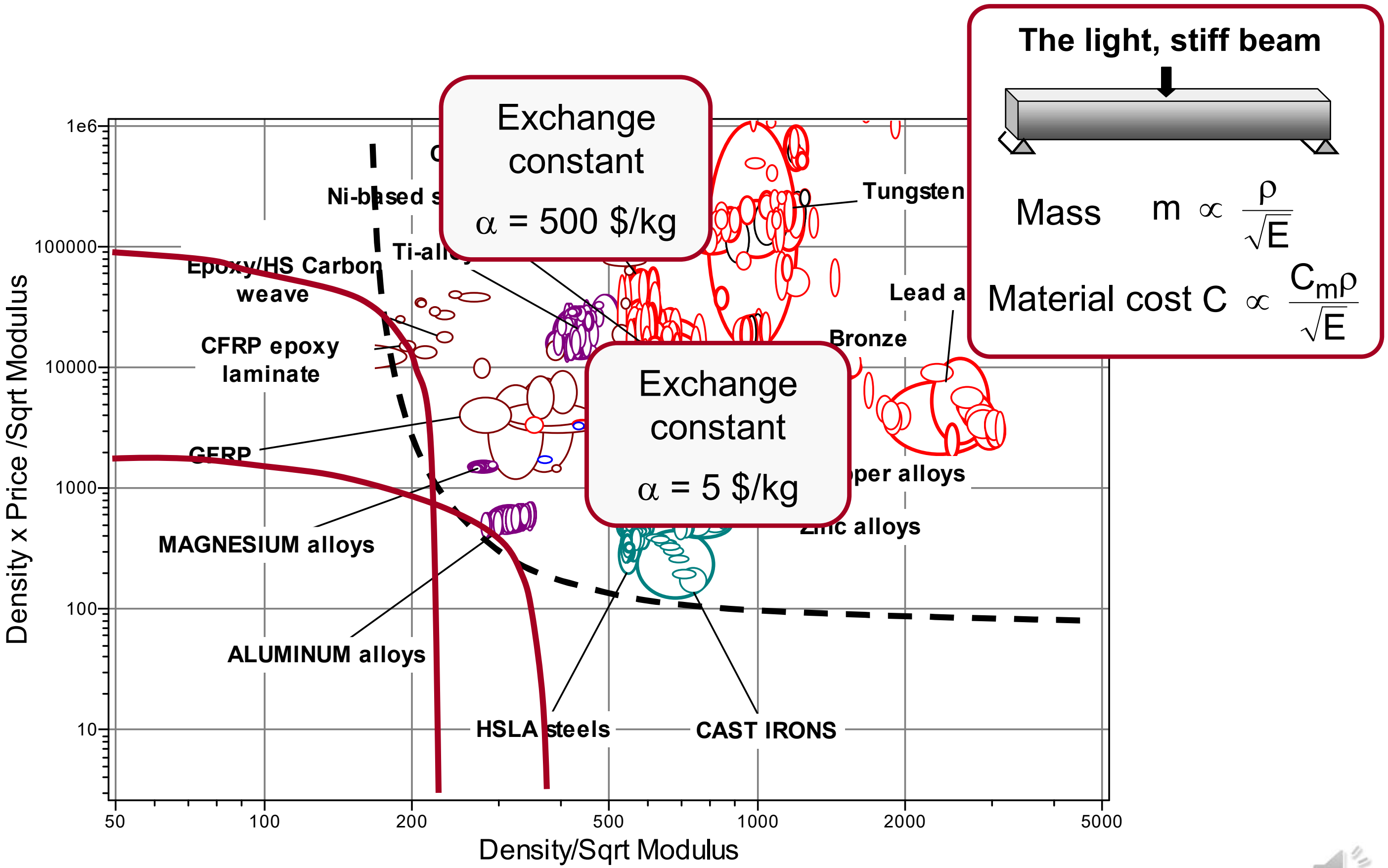
$$Z = \alpha m + C$$
$$C = -\alpha m + Z$$





All the material property charts have **logarithmic axes** – that is because material property-values span many decades. When a linear function (like the equation on the previous frame) is plotted on logarithmic axis, it appears as a curve, not a straight line. That is the only difference. The best compromise is still the one where the Z curve is tangent to the trade-off surface.

Trade off: mass vs. cost for given stiffness



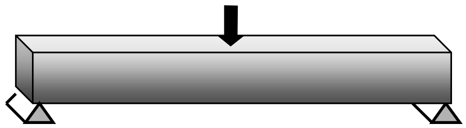
This is a trade-off plot for choosing materials for a stiff beam with **two objectives:**

- **Minimize mass**, m, proportional to $\rho/E^{1/2}$ (see Unit 4) and
- **Minimize material cost**, C, proportional to $C_m\rho/E^{1/2}$ where C_m is the cost of material per kg.

Countours of Z are shown for several values of Z, ranging from 0.5 $/kg to 500 $/kg. Each is tangent to the trade-off surface at a different point, the lowest at steels and cast irons, the highest at CFRP laminates.

Plotting the penalty function

The light, stiff beam



The penalty function is defined as

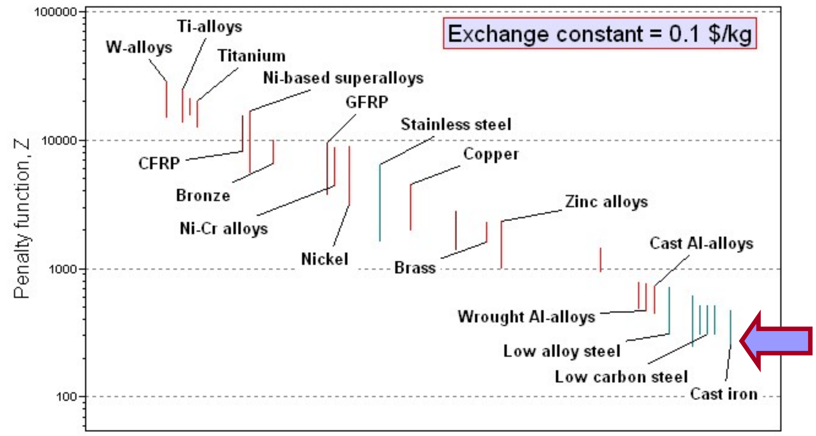
$$Z = \alpha m + C$$

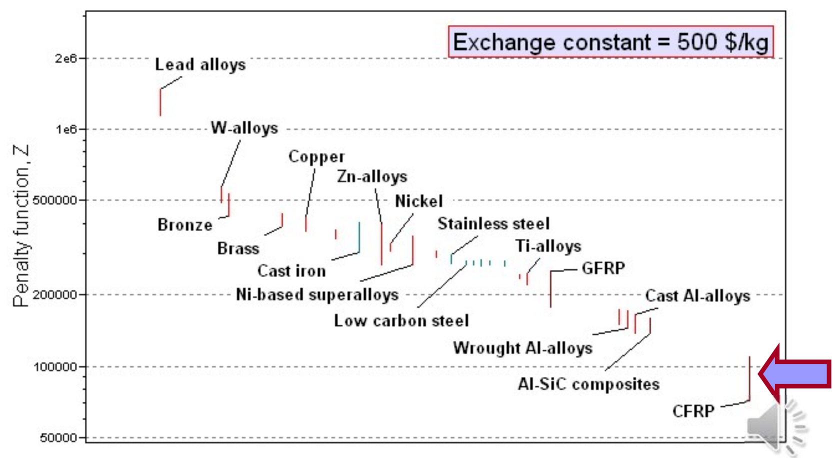
Mass $\quad m \propto \dfrac{\rho}{\sqrt{E}}$

Material cost $\quad C \propto \dfrac{C_m \rho}{\sqrt{E}}$

$$Z = \alpha \frac{\rho}{\sqrt{E}} + \frac{C_m \rho}{\sqrt{E}} = \frac{\rho}{\sqrt{E}} \left(C_m + \alpha \right)$$

Plot this for chosen α





The alternative (and the most effective) way of exploring trade-off is to plot the **penalty function Z as a whole**, using the **Advanced** facility in CES. Here it is written out for trade-off between weight and material cost for a light, stiff beam (for other problems, of course, it contains other combinations of material properties). On the right are two plots one for a low value of the exchange constant, one for a high (high means that mass carries a high penalty). For the low value (upper plot) cast irons and steels minimize Z. For the high (lower plot) it is minimized by CFRP and metal-matrix composites.

The main points

- Real design problems involve conflicting objectives -- often ***technical performance*** vs. ***economic performance*** (cost).

- **Trade-off plots** reveal the options, and (when combined with the other constraints of the design) frequently point to a final choice

- If the relative value of the two metrics of performance (measured by and **exchange constant**) is known, a **penalty function** allows an unambiguous selection

This unit has introduced ways of dealing with **conflicting objectives** in materials selection. The key concept is that of the **trade-off plot** – it alone is often enough to identify good choices. If greater precision is required, the **penalty function** method provides it.

Example: Multiple objectives: casing for a minidisk player

- Electronic equipment -- portable computers, players, mobile phones -- all miniaturised; many now less than 12 mm thick

- An ABS or Polycarbonate casing has to be > 1mm thick to be stiff enough for protection; casing occupies 20% of the volume

- Find best material for a stiff casing of minimum thickness and weight

Objective 1	minimise casing thickness
Objective 2	minimise casing mass

- The thinnest may not be the lightest … need to explore trade-off

Electronic devices – portable computers, mobile phones and players, PDA's – are getting smaller and lighter. Ideally they should slip into the pocket or the handbag without disarray to clothing. Many are now less than 12mm thick – the mini disk player shown here is an example. But although smaller, they must still sustain the same handling loads and survive the same shocks as the older, larger, equipment, requiring a casing of more or less the same strength. The usual ABS or polycarbonate casings have to be at least 1mm thick to be stiff enough – and that means that the casing takes up 20% or more of the available volume. The casing is a shell with broad, almost flat faces. When loaded these faces deflect inwards; if they deflect too much the display or the electronics are damaged. Generally it is this elastic deflection that is the problem, not the lack of strength – again a consequence of the thinness. The challenge is to find a better material for the casing, allowing a thinner product and, if possible, a lighter one.

Example: Multiple objectives: Performance metrics for the casing

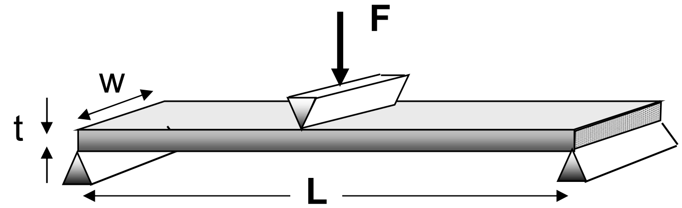
Function Stiff casing

Constraints

- Stiffness, S

$$S = \frac{48\,E\,I}{L^3} \quad \text{with} \quad I = \frac{w\,t^3}{12}$$

- Adequate toughness, $G_{1c} > 1\text{kJ/m}^2$



m = mass
w = width
L = length
ρ = density
t = thickness
S = required stiffness
I = second moment of area
E = Youngs Modulus

Objective 1 Minimise thickness t

Metric 1

$$t = \left(\frac{S L^3}{4 E w}\right)^{1/3} \propto \frac{1}{E^{1/3}}$$

Objective 2 Minimise mass m

Metric 2 (from Unit 2)

$$m = \left(\frac{12\,S\,w^2}{C}\right)^{1/3} L^2 \left(\frac{\rho}{E^{1/3}}\right) \propto \frac{\rho}{E^{1/3}}$$

This frame lays out the design requirements and develops equations for the two metrics of performance: thinness and low mass. The first equation defines the constraint: the stiffness. The stiffness of a flat panel of thickness t, width w, and length L is listed. Substituting for I and solving for t gives the first metric. The second – the mass of the panel for a given bending stiffness – we already derived in Unit 2. The equation is repeated here. There is, in addition, an obvious constraint of toughness. Minidisk players get dropped – a brittle material would shatter. We add the requirement of a toughness G1C > 1 kJ/m2.

Example: Multiple objectives: Relative performance metrics

- We are interested here in substitution. Suppose the casing is currently made of a material M_o.

- The thickness of a casing made from an alternative material M, differs (for the same stiffness) from one made of M_o by the factor

$$\frac{t}{t_o} = \left(\frac{E_o}{E}\right)^{1/3}$$

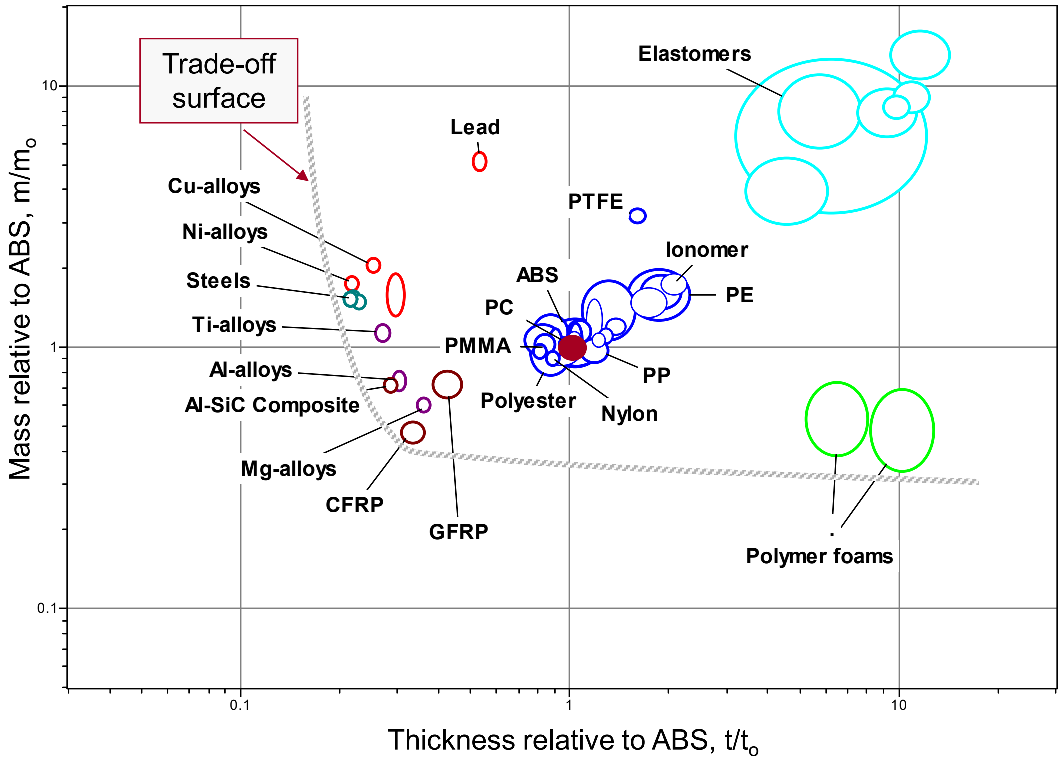
- The mass differs by the factor

$$\frac{m}{m_o} = \left(\frac{\rho}{E^{1/3}}\right) \cdot \left(\frac{E_o^{1/3}}{\rho_o}\right)$$

- Explore the trade-off between $\frac{t}{t_o}$ and $\frac{m}{m_o}$

We are interested here in substitution – in replacing the current ABS case with one that is thinner and lighter. Thus it is the factor by which these metrics change that is of interest – we don't need their absolute values. This greatly simplifies things. The frame lists the thickness t and mass m of a casing made of material M relative to the existing casing of thickness to and mass mo made of material Mo.

Example: Multiple objectives: The trade-off plot



Additional constraints:

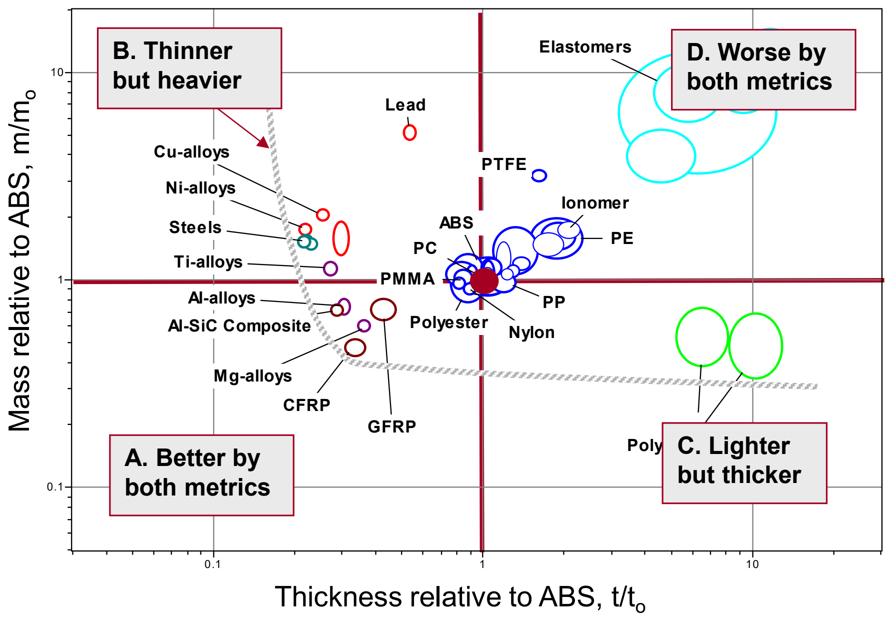
- G_{1c} > 1kJ/m^2
- **Wood, leather suppressed**.

- Finding a compromise: CFRP, Al and Mg alloys all offer reduction in mass and thickness

The chart shows relative thickness and relative mass of casings made from a range of materials. The two metrics have each been divided by the values for the currently used material, ABS, which therefore lies at the point (1,1). The requirement G1C>1kJ/m2 has been applied separately. The axes show the factor by which t and m change if the casing is made of an alternative material. Polymers are "dominated" solutions. The materials on the trade-off surface are metals or high-performance composites. If low weight is the dominant requirement, magnesium, CFRP and aluminium are good choices. If thinness is more important, then titanium and high strength steel are possible choices, although they are slightly heavier than ABS. Makers of electronic equipment have high-end models that use these materials – and they identify these in their advertising. Here they seek to enhance value not merely by exploiting the properties of the material, but by increasing the perceived value of the product.

Example: Multiple objectives: Postscript

- The four sectors of a trade-off plot for substitution

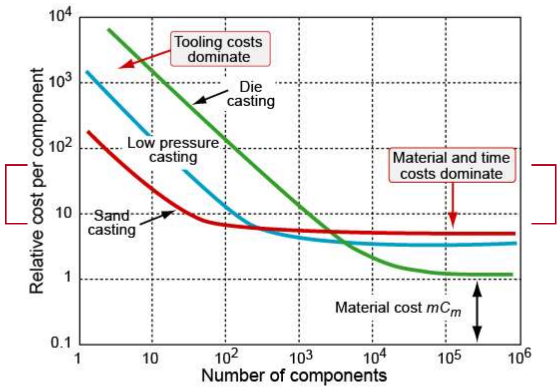


- Is material cost relevant? Probably not -- the case only weighs a few grams. Volume and weight are much more valuable.

The figure shows the trade-off plot again with four sectors marked. Sector A is the "winwin" sector – candidates here are both thinner and lighter than the existing casing. CFRP, aluminium and magnesium alloys and composites lie in this sector. Sectors B and C are "win-loose" sectors – lighter but thicker, or thinner but heavier. Sector D is uninteresting – candidates here are both heavier and thicker. Is material cost important? Not very: the casing only weighs a few grams; even if titanium were chosen the material cost is little more than £0.1 or 18 cents. The gain in performance more than offsets this.

The economics:

cost modelling for selection



Outline: estimating process cost

- Assessing potential: cost and value
- Inputs to a cost model for selection
- The model and its implementation
- Cost drivers, batch size, assembly

More info:
- "Materials: engineering, science, processing and design", **Chapter 18**
- "Materials Selection in Mechanical Design", **Chapters 7 and 8**

This Unit introduces simple ideas about cost modelling for material and process selection, and describes how they can be implemented

Cost, price and value

- **Cost** = what it actually costs to make the part or product
- **Price** = the sum you sell it for
- **Value** = the worth the consumer puts on the product

The real requirement is

Cost < Price < Value

C < P < V

To maximize profit, P - C we seek to minimize C

"Not worth the price" **means P > V**

"Good value for money"means **P < V**

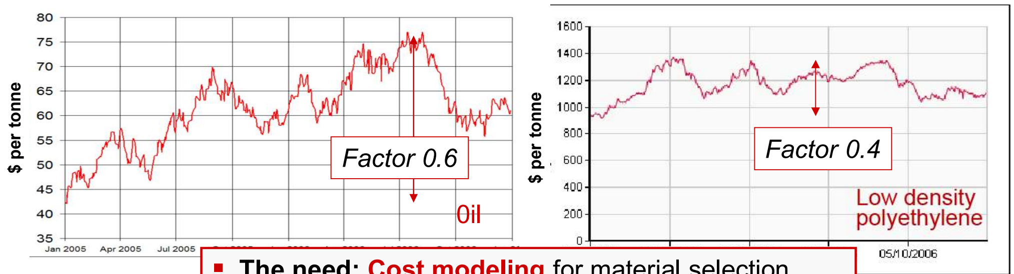
The cost of producing a component of or product is made up of

- the **material cost**
- the **cost of manufacture**

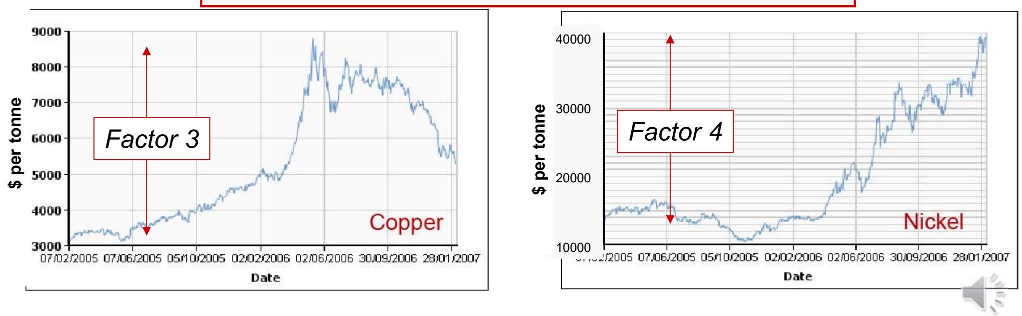
This first frame introduces the distinction between **cost C, price P** and **value V**. Materials and processes are chosen to maximize value and minimize cost, giving the greatest scope for **profit P – C**. This is achieved by minimizing material and manufacturing cost without compromising quality.

The problem of material price

- **Changing price of materials 2005 - 2007**



- **The need: Cost modeling** for material selection



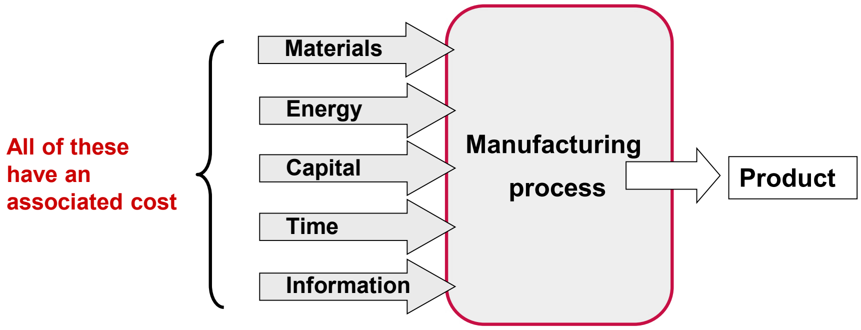
Material prices fluctuate. In the period 2005 – 2007 the price of copper (thus brass and bronze), nickel (thus stainless steels), zinc (thus galvanized sheet, used for car panels) and lead (batteries) all changed by large factors. Commodity polymers changed much less, despite fluctuations in oil price.

Estimating cost

When alternative material-process combinations meet the constraints, it is logical to rank them by **cost**

- **Cost estimate for competitive bidding --** *absolute* cost is wanted, to ± **5%**
- **Cost estimate for ranking --** a *relative* cost is OK – but need generality

Generic inputs to any manufacturing process:



The nature and detail of cost modelling depends on the purpose for which it is done. Cost estimation for competitive bidding for a contract is a skilled job: an error of 5 % can mean the difference between profit and loss. Our purpose here is quite different: it is to estimate **relative cost** with just enough precision to **compare competing processes.**

The manufacture of a component consumes resources, shown in the lower part of this frame. Each has an associated cost. The final cost is the sum of those of the resources it consumes. They are defined in the next frame.

Inputs to a generic cost estimator

Generic = can be applied to any process

Resource		Symbol	Unit
Materials including consumables		C_m	\$/kg
Capital	cost of equipment cost of tooling	C_c C_t	\$ \$
Time (including labor) overhead rate		$\dot{C}_{oh}$	\$/hr
Energy cost of energy		C_e	\$/hr
Space, **admin.** a cost/hr		$\dot{C}_{s,a}$	\$/hr
Information	R & D royalties, licenses	$\dot{C}_i$	\$/hr

Lump into overhead rate $\dot{C}_{oh}$

The cost of producing a component of mass m entails:

- the cost C_m (\$/kg) of the **materials** and **consumable feed-stocks** from which it is made.
- the cost of dedicated **tooling**, C_t (\$)
- the cost of the capital **equipment**, C_c (\$), in which the tooling will be used.
- the cost of **time**, chargeable at an overhead rate $\dot{C}_{oh}$ (thus with units of \$/hr), in which we include the cost of labor, administration and general plant costs.
- the cost of **energy**, C_e which is sometimes charged against a process-step if it is very energy intensive but more usually is treated as part of the overhead and lumped into $\dot{C}_{oh}$, as we shall do here.
- the cost of **information**, meaning that of research and development, royalty or license fees; this, too, we view as a cost per unit time and lump it into the overhead.

The cost per unit of output

Material costs C_m per kg, and a mass m is used per unit; **f** is the scrap fraction (the fraction thrown away) $\Rightarrow \dfrac{m C_m}{1-f}$

Tooling C_t is "dedicated" -- it is written off against the number of parts to be made, **n** $\Rightarrow \dfrac{C_t}{n}$

Capital cost C_c of equipment is "non-dedicated"
It is written off against time, giving an hourly rate.
The write-off time is t_{wo}. The rate of production is $\dot{n}$ units/hour.
The load factor (fraction of time the equipment is used) is **L**. $\Rightarrow \dfrac{1}{\dot{n}}\left(\dfrac{C_c}{L.t_{wo}}\right)$

The gross **overhead rate** $\dot{C}_{oh}$ contributes a cost per unit of time that, like capital, depends on production rate $\dot{n}$ $\Rightarrow \dfrac{\dot{C}_{oh}}{\dot{n}}$

Materials; Tooling; Capital, Labor, Information, Energy...

$$C = \left[\frac{m\,C_m}{1-f}\right] + \left[\frac{\sum (C_t)}{n}\right] + \frac{1}{\dot{n}}\left[\sum\left(\frac{C_c}{L\,.\,t_{wo}}\right) + \dot{C}_{oh}\right]$$

Batch size; Rate of production

Consider now the manufacture of a component (the "unit of output") weighing *m* kg, and made of a material costing C_m \$/kg. The first contribution to the unit cost is that of the **material** mC_m magnified by the factor *1/(1-f)* to account for the fraction that is lost.

The cost C_t of a set of **tooling** – dies, molds, fixtures and jigs – is what is called a *dedicated cost*: one that must be wholly assigned to the production run of this single component. It is written off against the numerical size *n* of the production run, giving the second term in this frame

The capital cost of **equipment**, C_c, by contrast, is rarely dedicated. A given piece of equipment – a powder press, for example – can be used to make many different components by installing different die-sets or tooling. It is usual to convert the capital cost of *non-dedicated* equipment and the cost of borrowing the capital itself into an overhead by dividing it by a *capital write-off time*, t_{wo}, (5 years, say) over which it is to be recovered. The quantity C_c/t_{wo} is then a cost per hour – provided the equipment is used continuously. That is rarely the case, so the term is modified by dividing it by a *load factor*, *L* – the fraction of time for which the equipment is productive. This gives an effective hourly cost of the equipment, like a rental charge. This gives the third term above.

Finally there is the general background hourly **overhead rate** for labor, energy and so on $\dot{C}_{oh}$. This is again converted to a cost per unit by dividing by the production rate $\dot{n}$ units per hour, giving the fourth term.

The **total cost** per part, C_s, is the sum of these four terms, C_1 to C_4, giving the final equation.

This establishes the bare bones of a tool for estimating the relative cost producing a unit of output. It can be refined in many ways. The CES software has a slightly more refined version, implemented for shaping processes

The cost per unit of output

Material costs C_m per kg, and a mass m is used per unit; **f** is the scrap fraction (the fraction thrown away) $\Rightarrow \dfrac{mC_m}{1-f}$

Tooling C_t is "dedicated" -- it is written off against the number of parts to be made, **n** $\Rightarrow \dfrac{C_t}{n}$

Capital cost C_c of equipment is "non-dedicated"
It is written off against time, giving an hourly rate.
The write-off time is t_{wo}. The rate of production is $\dot{n}$ units/hour.
The load factor (fraction of time the equipment is used) is **L**. $\Rightarrow \dfrac{1}{\dot{n}}\left(\dfrac{C_c}{L.t_{wo}}\right)$

The gross **overhead rate** $\dot{C}_{oh}$ contributes a cost per unit of time that, like capital, depends on production rate $\dot{n}$ $\Rightarrow \dfrac{\dot{C}_{oh}}{\dot{n}}$

$$C = \left[\frac{m\,C_m}{1-f}\right] + \left[\frac{\sum(C_t)}{n}\right] + \frac{1}{\dot{n}}\left[\sum\left(\frac{C_c}{L.t_{wo}}\right) + \dot{C}_{oh}\right]$$

Materials; Tooling; Capital, Labor, Information, Energy...; Batch size (n); Rate of production ($\dot{n}$)

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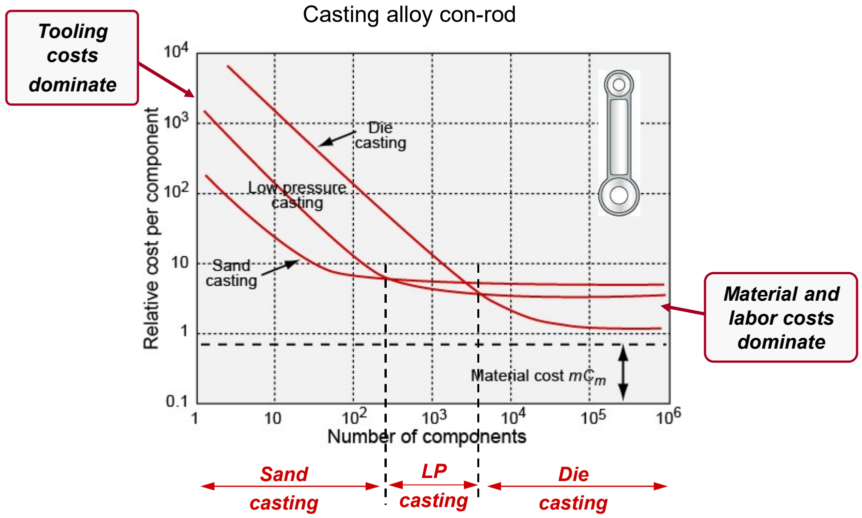
Features of a cost model



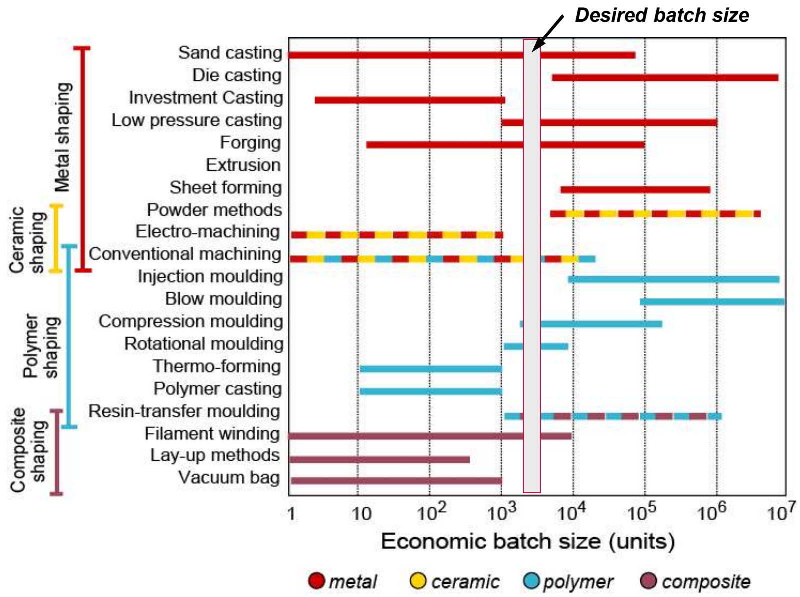
- Identify most economic process
- Examine materials-cost sensitivity
- Explore alternative materials and processes

The equation on the last frame is simplified here by lumping the terms together to give the three shown here. This equation says: the cost has three essential contributions – a **material cost per unit of production** that is independent of batch size and rate, a **dedicated cost per unit of production** that varies as the reciprocal of the production volume *(1/n),* and a **gross overhead per unit of production** that varies as the reciprocal of the production rate ( ). The dedicated cost, the effective hourly rate of capital write-off and the production rate can all be defined by a representative range for each process; target batch size , the overhead rate , the load factor and the capital write-off time must defined by the user.

The figure is a plot of cost, *C* , against batch size, *n*, comparing the cost of casting a small aluminum component by three alternative processes: sand casting, die casting and low pressure casting. At small batch sizes the unit cost is dominated by the "fixed" costs of tooling (the second term on the right of the equation). As the batch size *n* increases, the contribution of this to the unit cost falls (provided, of course, that the tooling has a life that is greater than *n*) until it flattens out at a value that is dominated by the "variable" costs of material, labour and other overheads. Competing processes differ in tooling cost C_t and equipment cost C_c and production rate $\dot{n}$. Sand casting equipment is cheap but slow. Die casting equipment costs much more but is also much faster. Mold costs for low pressure die casting are greater than for sand casting, those for high pressure die casting are higher still. The combination of all these factors for each process causes the $C_s - n$ curves to cross, as shown in the figure.

The cross-over means that the process that is cheapest depends on the batch size. This suggests the idea of an **economic batch size** – a range of batches for which each process is likely to be the most competitive. The equation on the earlier frames allows the cost of competing processes to be compared if data for the parameters of the model are known. If they are not, the economic batch size provides and alternative way of ranking.

Economic batch size



This is a bar chart of economic batch size for a number of common processes color coded by material. Processes such as investment casting of metals and lay-up methods for composites have low tooling costs but are slow; they are economic when you want to make small number of components but not when you want a large one. The reverse is true of the die casting of metals and the injection molding of polymers: they are fast, but the tooling is expensive.

Where do you get the input information?

- **Material and process costs** vary with time and depend on the quantity you order
- CES has approximate cost for 2900 materials and 80 processes
- Web helps with commodity materials
  - American Metal Market On-line, www.amm.com
  - Iron & Steel Statistics Bureau, www.issb.co.uk
  - Kitco Inc Gold & Precious Metal Prices, www.kitco.com/gold.live.html - ourtable
  - London Metal Exchange, www.lme.co.uk
  - Metal Bulletin, www.metalbulletin.plc.uk
  - Mineral-Resource, minerals.usgs.gov/minerals
  - The Precious Metal and Gem Connection, www.thebulliondesk.com/default.asp
- Ask suppliers: but how find them?
  - Thomas Register of European Manufacturers, TREM
  - Thomas Register of North American Manufacturers
  - Kelly's register

Getting data about cost is difficult. For the purposes of comparison (out purpose here), approximated data are often adequate. The CES software has approximate cost data for materials and, for shaping processes, uses the cost model described in earlier frames. To get further it is essential to ask the material and process suppliers. They can be located using free **Registers**, annually updated, like those listed here.

Cost modelling in CES

$$C = \left[\frac{m\,C_m}{1-f}\right] + \left[\frac{C_t}{n}\right] + \frac{1}{\dot{n}}\left[\frac{C_c}{L\,.\,t_{wo}} + \dot{C}_{oh}\right]$$

Characteristics of the process

Cost of equipment	C_c
Cost of tooling	C_t
Production rate	$\dot{n}$

The database has approximate value-ranges for these

Site-specific, user defined parameters

Batch size	n
Mass of component	m
Capital write-off time	t_{wo}
Load factor	L
Overhead rate	$\dot{C}_{oh}$

These are entered by the user via a dialog box

The CES software includes the **batch-process cost model**. A dialog box allows the user to edit default values of the user-defined parameters etc. The software then retrieves approximate values for the economic process attributes from the database where they are stored as ranges. It allows the data to be presented in a number of ways, two of which are shown in the next two frames.

Cost modelling in CES

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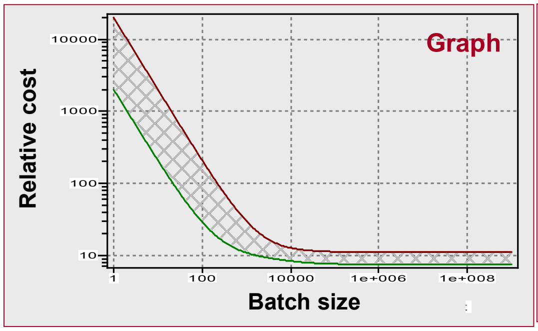
Cost model in CES Levels 2 and 3

Cost modelling

Relative cost index (per unit)	5	-	6	
Capital cost	2000	-	5000	GBP
Material utilisation factor	0.7	-	0.75	
Production rate (units)	20	-	30	per hr.
Tooling cost	300	-	450	GBP
Tooling life	5000	-	10000	units



Dialog box

Capital write-off time t_{wo} = ...

Component mass m = ...

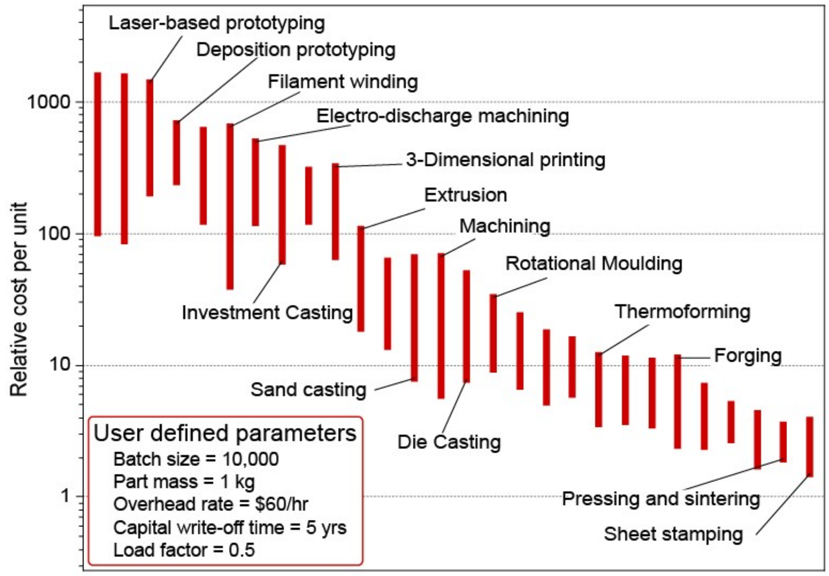
Load factor L = ...

Material cost Cm =

Overhead rate $\dot{C}_{oh}$ = ...

This is one way in which cost data can be plotted in CES. It is a graph of cost against batch size for a single process, here injection molding, in the manner of the earlier figure. The user-defined parameters are listed on it. The band width derives from the ranges of the economic attributes: a simple shape, requiring only simple dies, lies near the lower edge; a more complex one, requiring multi-part dies, lies near the upper edge.

Cost model in CES Levels 2 and 3



This frame shows an alternative presentation. Here the range of cost for making a chosen batch size (here, 10,000) of a component by a number of alternative processes is plotted as a bar chart. The user-defined parameters are again listed. Other selection stages can be applied in parallel with this one applying constraints on material, shape, etc. causing some of the bars to drop out. The effect is to rank the surviving processes by cost.

The main points

- To maximize profit: **minimize cost C** (economics of manufacture) and **maximize value V** (technical performance and product image)
- **Cost can be modeled** at several levels -- depends on purpose
- To **rank** process options, approximate modeling is adequate
- A **cost-model** for this uses "generic" inputs: material, time, capital etc
- More precise analysis must be based on information from suppliers or (if out-sourcing) contractors.

This frame summarizes the main points.

Appendix

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